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Neutron Physics Division

GRAPHS OF SCATTERED PARTICLE ENERGIES VS SCATTERING ANGLES, AND GRAPHS OF CENTER-OF-MASS SCATTERING ANGLES VS LABORATORY SCATTERING ANGLES FOR NUCLEON-NUCLEON REACTIONS FROM 25 TO 3500 MeV AND FOR PION-NUCLEON REACTIONS FROM 25 TO 2500 MeV

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Abstract

Graphs are presented illustrating the laboratory energy of the scattered nucleon vs the laboratory energy of the incident nucleon where the scattering angle is a parameter for incident nucleon energies ranging from 25 to 3500 MeV. The same type of data is presented for pion-nucleon reactions with energies from 25 to 2500 MeV. Graphs of the scattering angle in the center-of-mass system vs the scattering angle in the laboratory system at various reaction energies for the same reactions and over the same energy ranges are presented. The data are applicable to the checking and the understanding of the results from an intermediate-energy intranuclear-cascade calculation.

NOTE:

This Work Partially Supported by
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Under Order R-104(1)

The need for the data in this report has occasionally risen while debugging an intermediate-energy intranuclear-cascade calculation, and while attempting to understand the energy and angular dependence of the results when comparisons with experiment are made. It was decided to calculate these quantities by computer in order to cover the energy and angular range of interest.

All of the energies and angles illustrated in Figs. 1 - 11 are laboratory system values.

Figures 1 - 3 illustrate the energy of the scattered nucleon vs incident nucleon energy in nucleon-nucleon scattering reactions ranging from 25 to 3500 MeV where the scattering angle is a parameter.

Figures 4 - 11 illustrate the energy of the scattered pion vs incident pion energy in pion-nucleon scattering reactions ranging from 25 to 2500 MeV using the scattering angle as a parameter.

Figures 12 - 22 are graphs of the center-of-mass scattering angle vs the laboratory scattering angle for nucleon-nucleon scattering reactions at various energies from 25 to 3500 MeV.

Figures 23 - 31 contain similar data, i.e., plots of the center-of-mass scattering angle vs laboratory scattering angle for pion-nucleon scattering reactions from 25 to 2500 MeV.

The expressions from which the data above were calculated are relativistic. It is assumed that the struck particle is at rest. The notation is defined as follows:

T_i = Kinetic energy (MeV) of the incident particle in the laboratory system

T_s = Kinetic energy (MeV) of the scattered particle in the laboratory system

v = Velocity of the center-of-mass system (in units of C)

p_s = The momentum (MeV/C) of the scattered particle, in the laboratory system

m_n = The nucleon rest mass in MeV

m_π = The pion rest mass in MeV

E = The total energy of the particle, $E = T + m$

θ = The scattering angle in the laboratory system

θ' = The scattering angle in the center-of-mass system.

The following expressions were derived using straightforward relativistic kinematics.

The energy of a nucleon, scattered at an angle θ, for a nucleon-nucleon scattering reaction of energy, T_i, is given by

$$T_s = \frac{T_i \cos^2\theta}{1 + \frac{T_i \sin^2\theta}{2m_n}} \qquad (1)$$

The energy of a pion, scattered at an angle θ, for a pion-nucleon scattering reaction where the incident pion energy is T_i, is

$$T_s = \{[-(E_i + m_n)(2E_i m_n + 2m_\pi^2) \pm \{(E_i + m_n)^2 (2E_i m_n + 2m_\pi^2)^2 + [(E_i^2 - m_\pi^2)\cos^2\theta - (E_i + m_n)^2] [(2E_i m_n + 2m_\pi^2)^2 + 4(E_i^2 - m_\pi^2) \times m_\pi^2 \cos^2\theta]\}^{1/2}]/2[(E_i^2 - m_\pi^2)\cos^2\theta - (E_i + m_n)^2]\} - m_\pi \qquad (2)$$

where the minus sign in front of the square root is used for $\theta \leq 90^\circ$ and the plus sign is used for $\theta \geq 90^\circ$.

The scattering angle in the center-of-mass system, θ', in terms of laboratory system quantities is

$$\theta' = \arctan\left[\frac{\sqrt{1-v^2}\,\sin\theta}{\cos\theta - \frac{vE}{p}}\right] \tag{3}$$

For nucleon-nucleon reactions

$$v = \frac{\sqrt{T_i^2 + 2T_i m_n}}{T_i + 2m_n}$$

$$E = T_s + m_n \qquad \text{where } T_s \text{ is given by Eq. (1)}$$

$$p = \sqrt{T_s^2 + 2T_s m_n}$$

For pion-nucleon reactions

$$v = \frac{\sqrt{T_i^2 + 2T_i m_\pi}}{T_i + m_\pi + m_n}$$

$$E = T_s + m_\pi \qquad \text{where } T_s \text{ is given by Eq. (2)}$$

$$p = \sqrt{T_s^2 + 2T_s m_\pi}$$

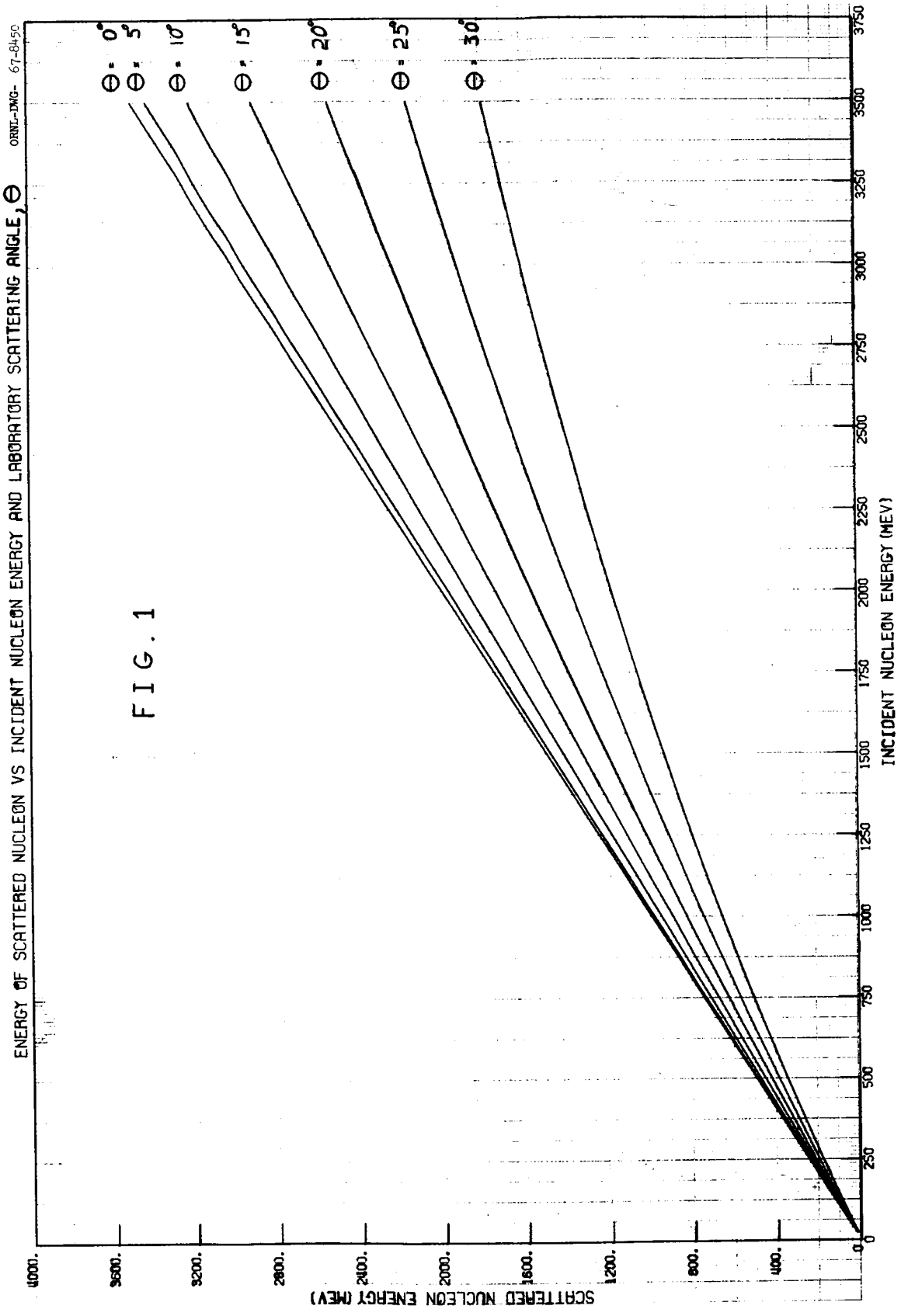

ENERGY OF SCATTERED NUCLEON VS INCIDENT NUCLEON ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG- 67-8450
Θ = 0°
Θ = 5°
Θ = 10°
Θ = 15°
Θ = 20°
Θ = 25°
Θ = 30°
SCATTERED NUCLEON ENERGY (MEV)
INCIDENT NUCLEON ENERGY (MEV)
0
400.
800.
1200.
1600.
2000.
2400.
2800.
3200.
3600.
4000.
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000
3250
3500
3750

FIG. 1

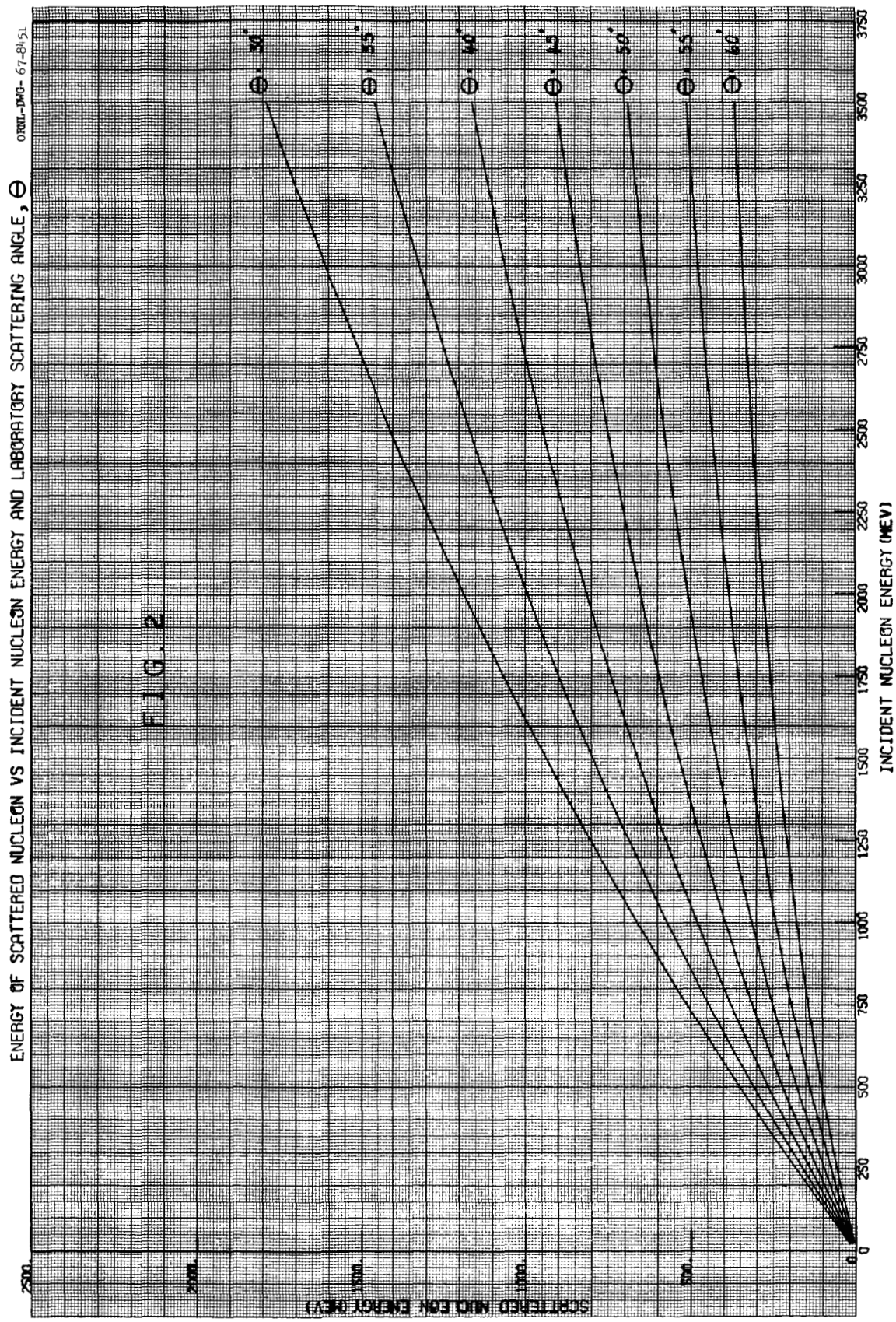

ENERGY OF SCATTERED NUCLEON VS INCIDENT NUCLEON ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG- 67-8451
SCATTERED NUCLEON ENERGY (MEV)
INCIDENT NUCLEON ENERGY (MEV)
Θ 30°
Θ 35°
Θ 40°
Θ 45°
Θ 50°
Θ 55°
Θ 60°
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000
3250
3500
3750
500
1000
1500
2000
2500

FIG. 2

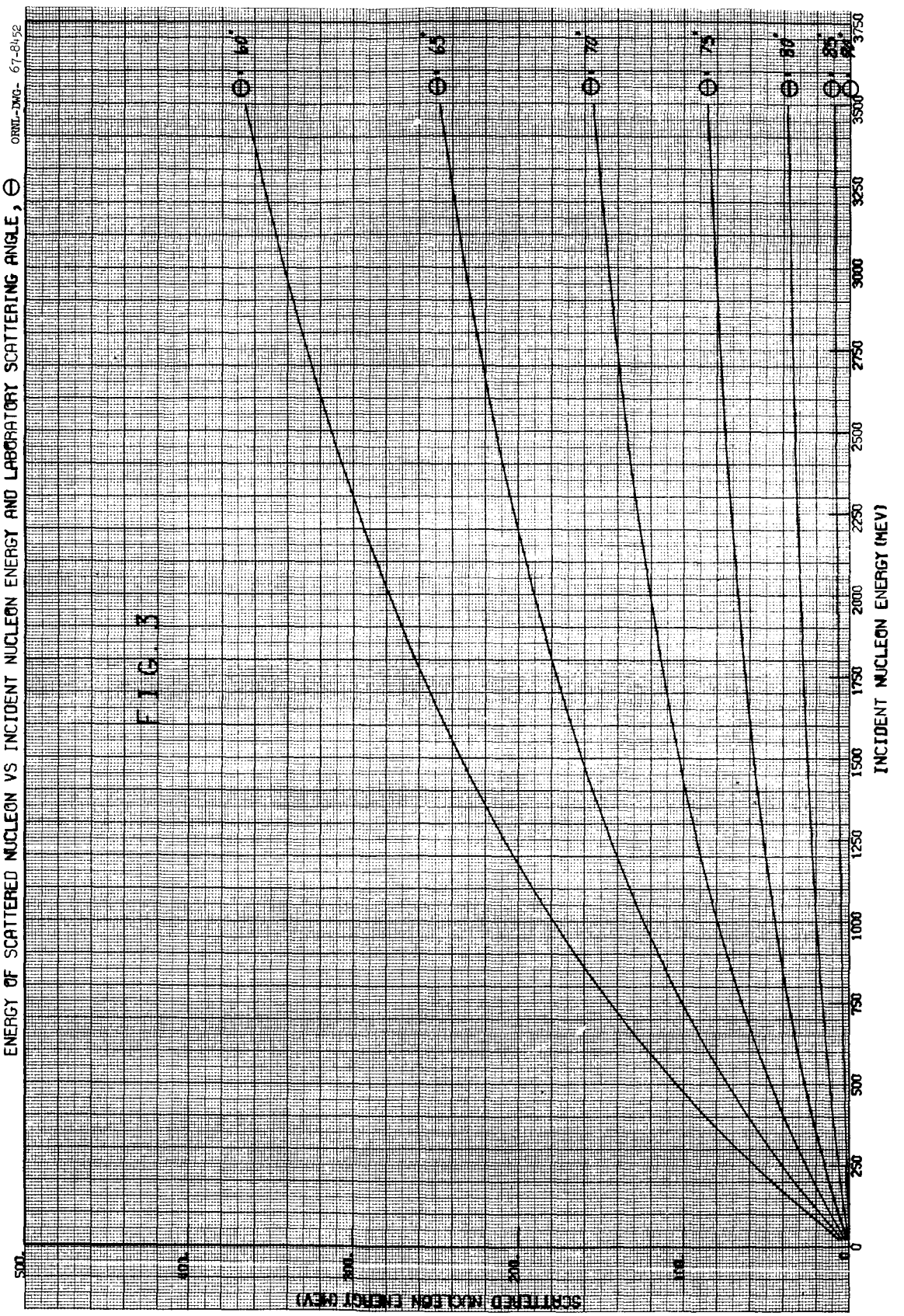

ENERGY OF SCATTERED NUCLEON VS INCIDENT NUCLEON ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG- 67-8452
SCATTERED NUCLEON ENERGY (MEV)
INCIDENT NUCLEON ENERGY (MEV)
Θ = 45°
Θ = 60°
Θ = 75°
Θ = 90°
Θ = 105°
Θ = 120°
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000
3250
3500
3750

FIG. 3

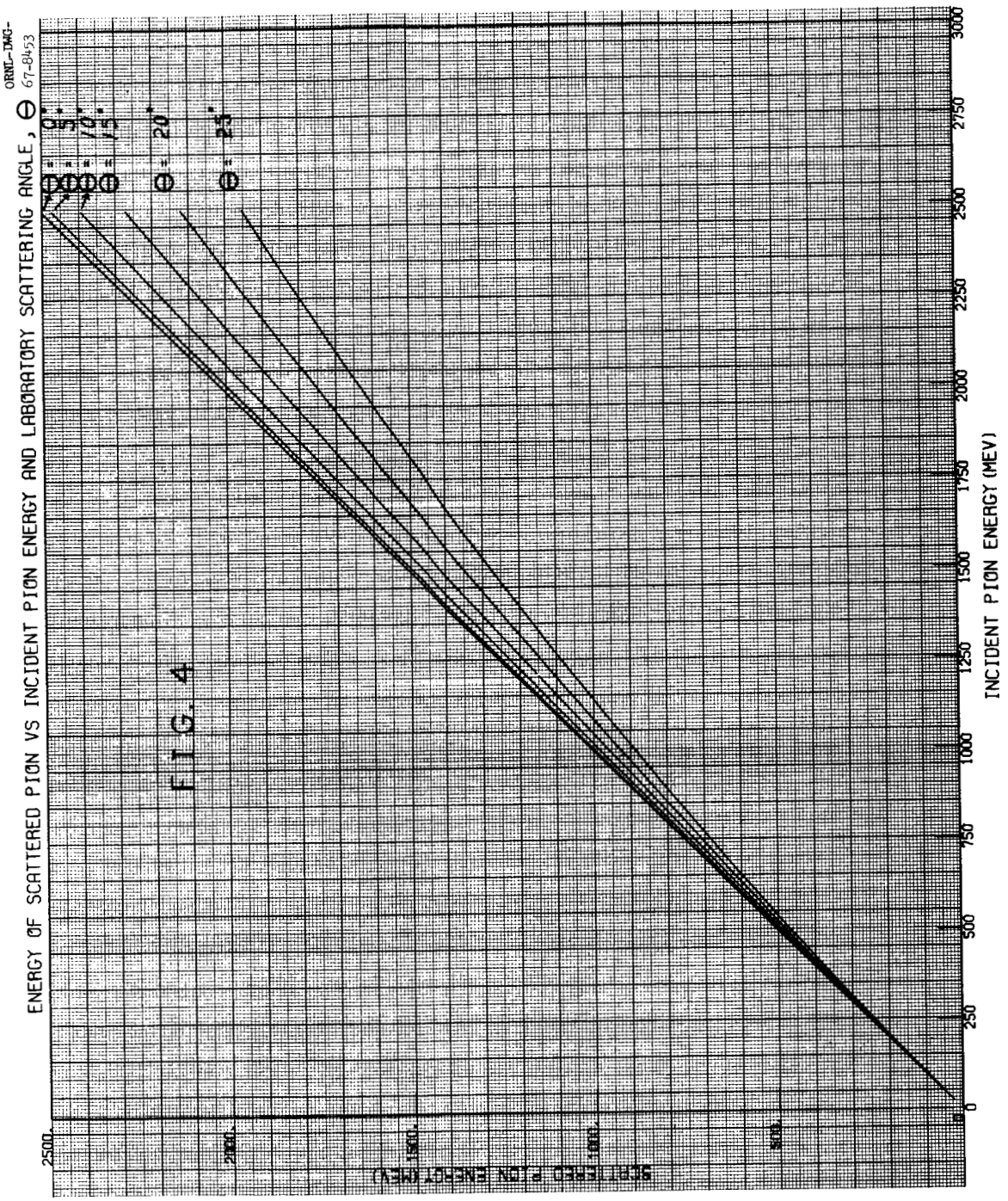

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG-67-8453
Θ = 5°
Θ = 10°
Θ = 15°
Θ = 20°
Θ = 25°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000

FIG. 4

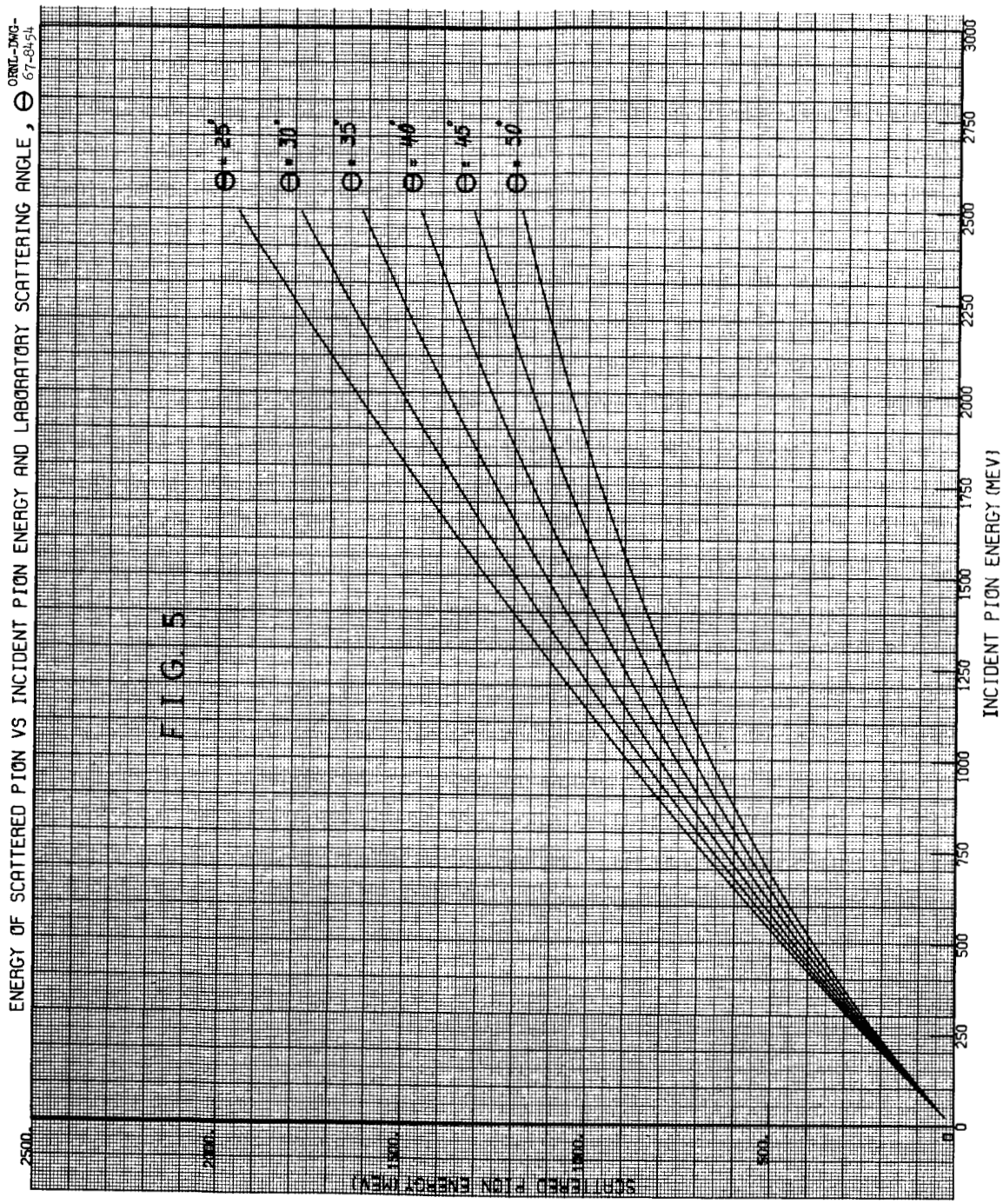

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG-67-8454
Θ = 25°
Θ = 30°
Θ = 35°
Θ = 40°
Θ = 45°
Θ = 50°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000

FIG. 5

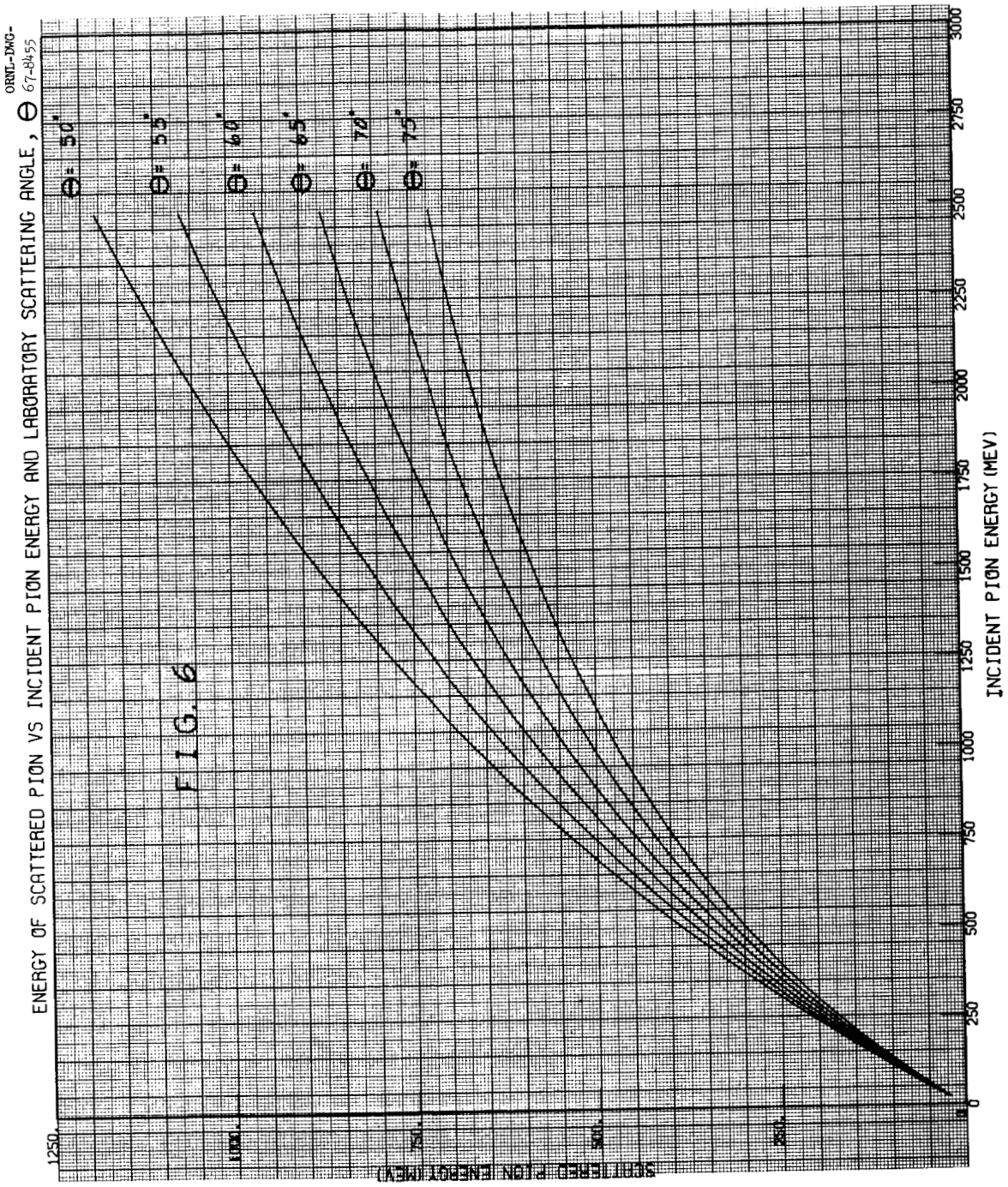

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG-67-8455
Θ= 5.0°
Θ= 5.5°
Θ= 6.0°
Θ= 6.5°
7.0°
7.5°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
1250
1000
750
500
250
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000

FIG. 6

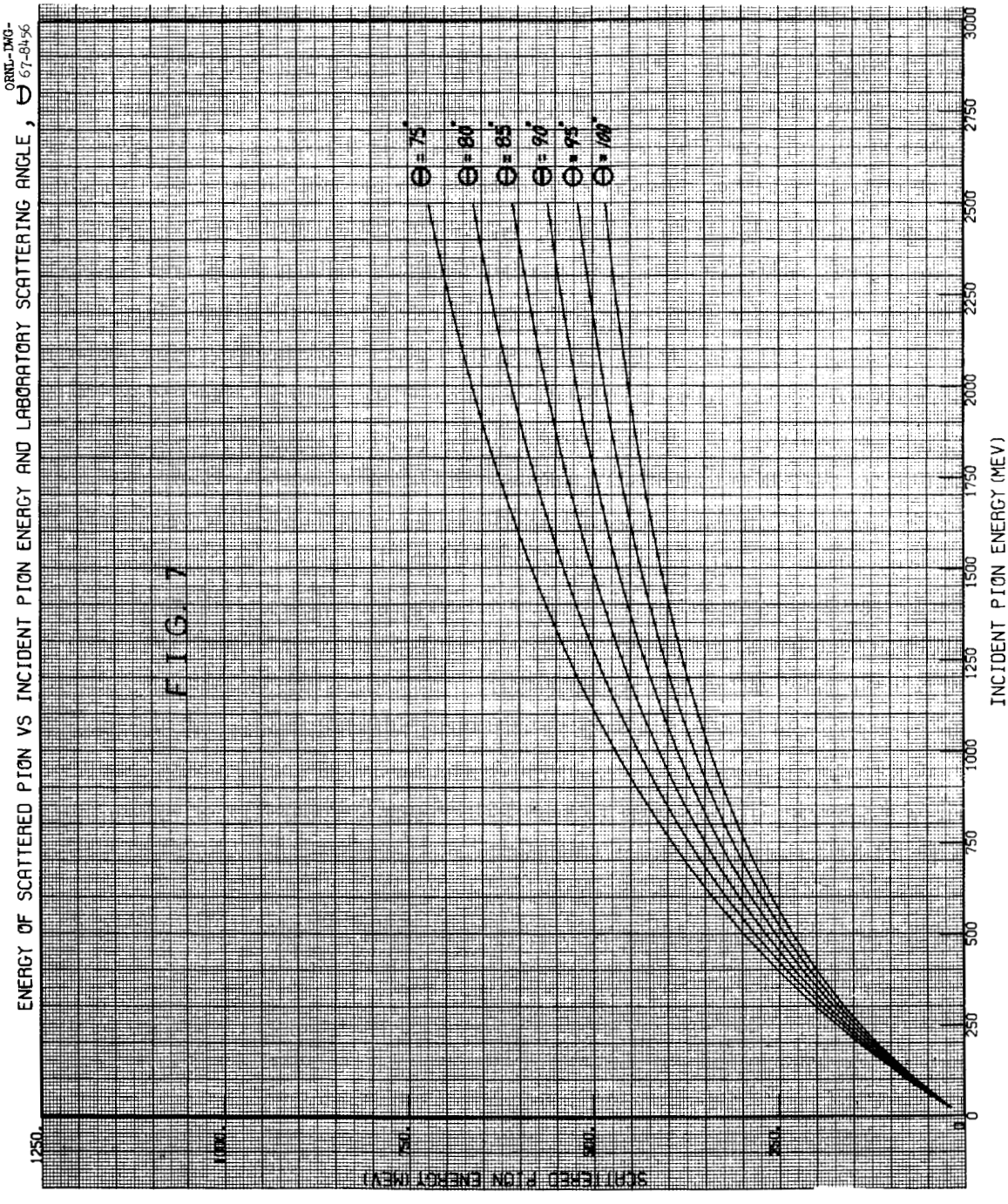

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, θ
ORNL-DWG-67-8496
θ = 75°
θ = 80°
θ = 85°
θ = 90°
θ = 95°
θ = 100°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000

FIG. 7

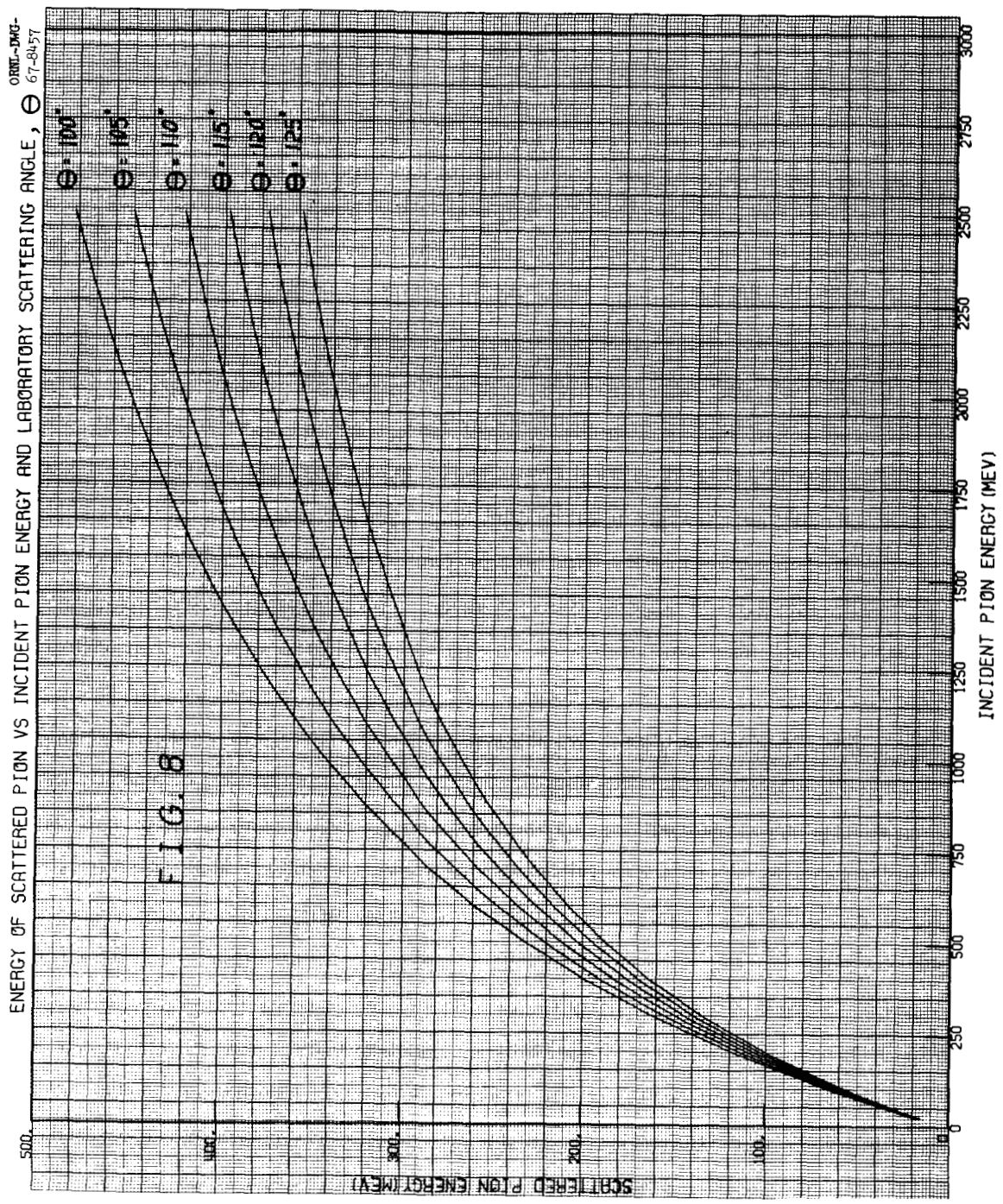

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG-67-8457
Θ = 90°
Θ = 105°
Θ = 110°
Θ = 115°
Θ = 120°
Θ = 125°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
0
100
200
300
500
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000

FIG. 6

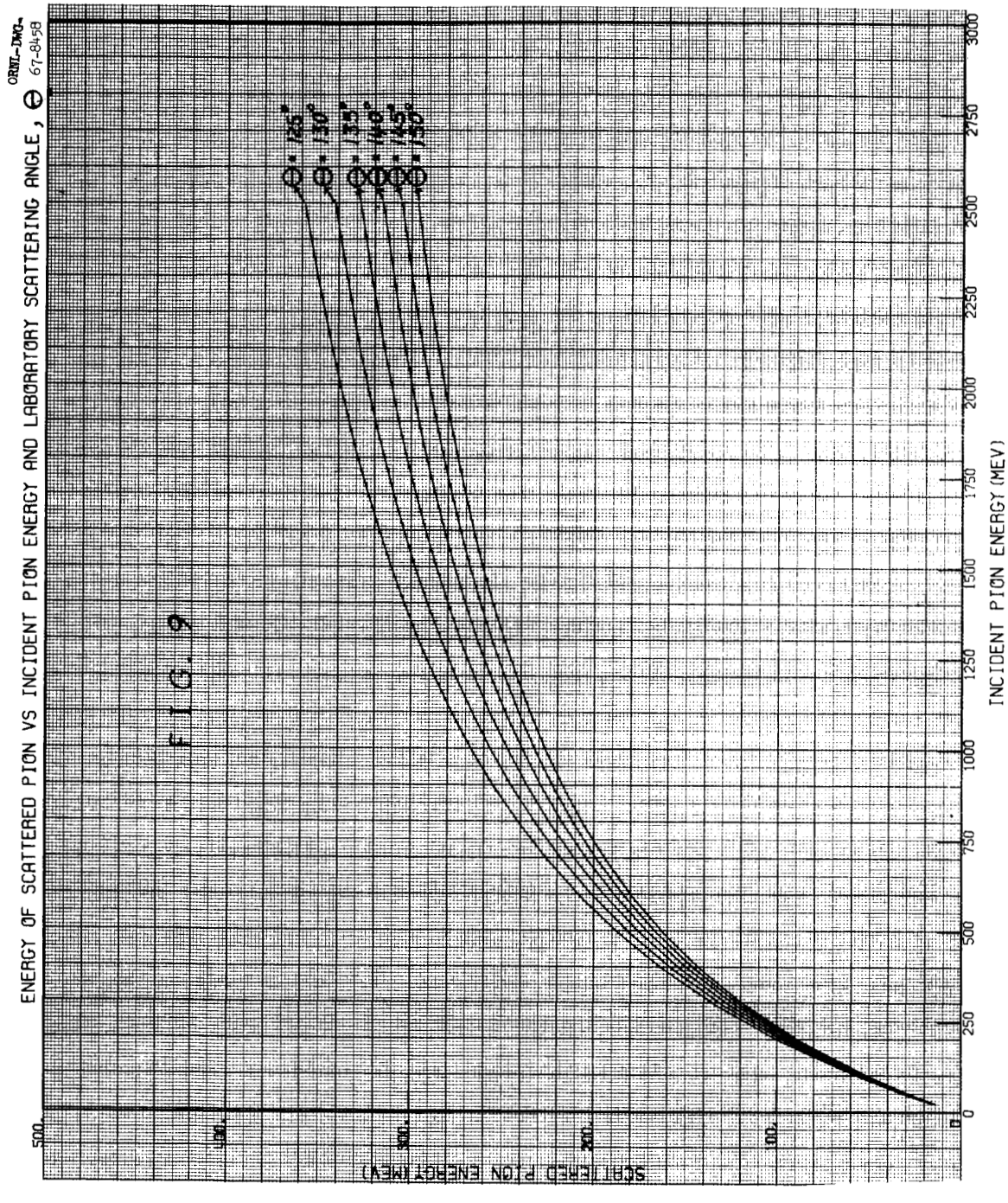

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, θ
ORNL-DWG-67-8458
25°
35°
45°
90°
135°
150°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000
100
200
300
500

FIG. 6

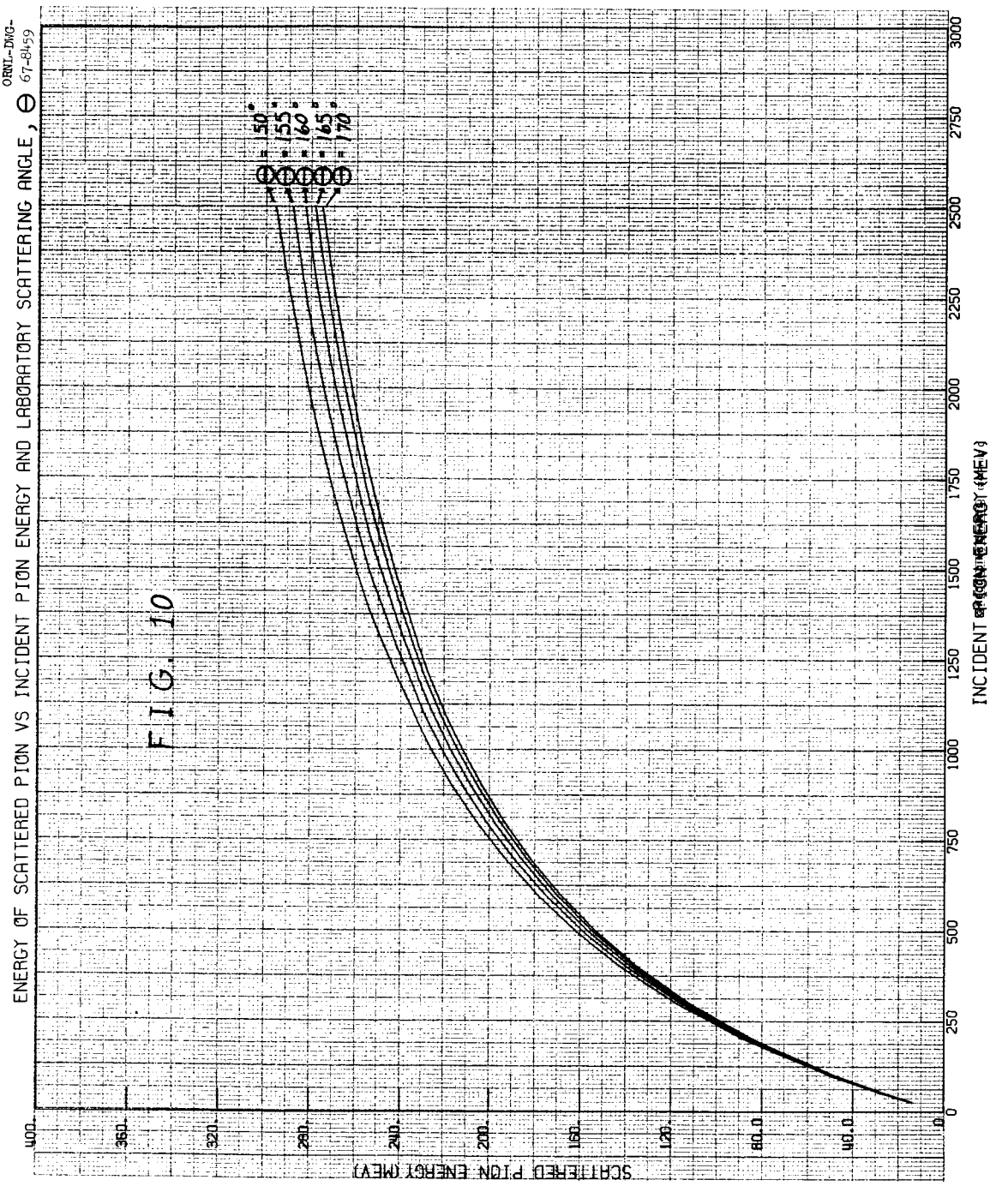
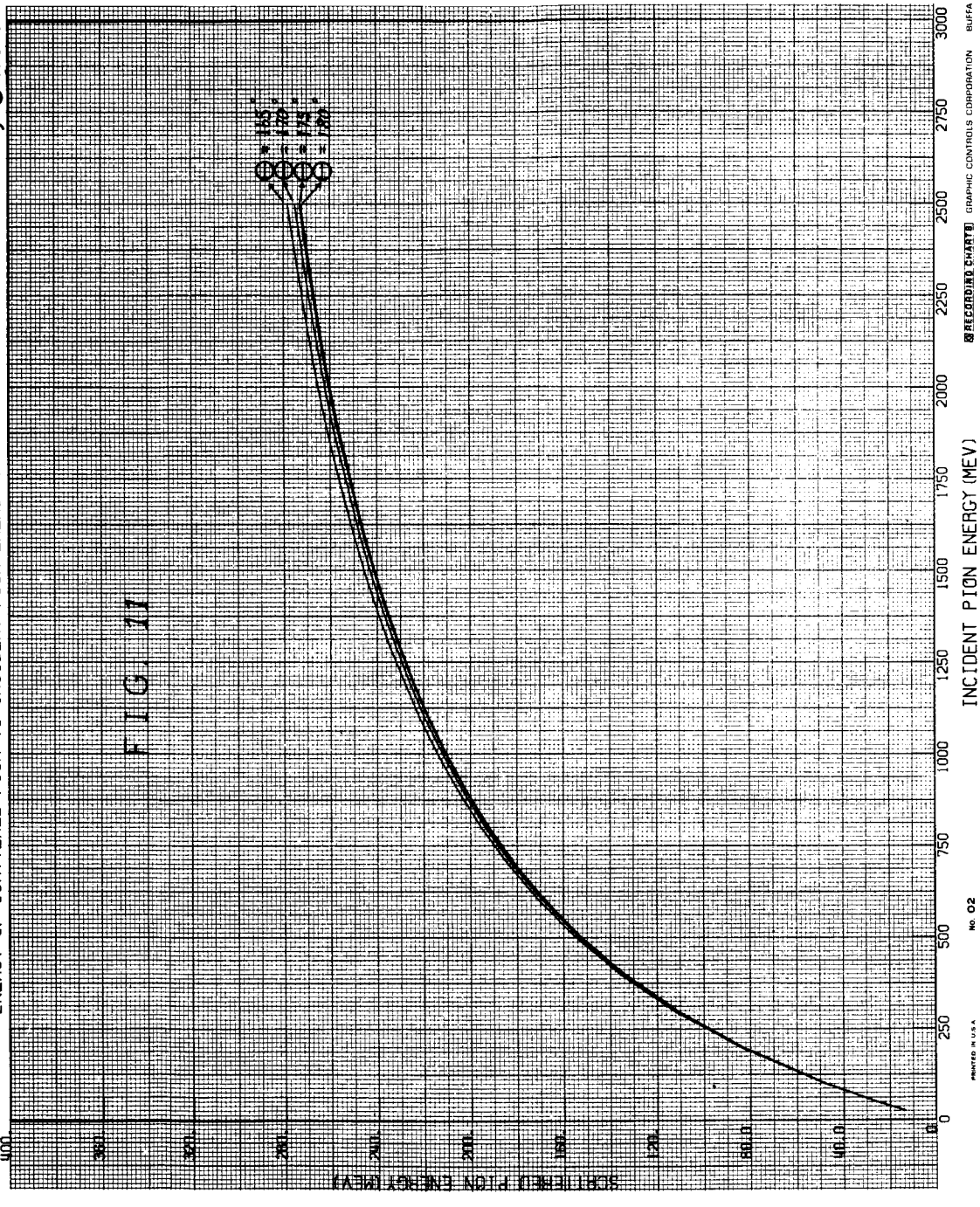
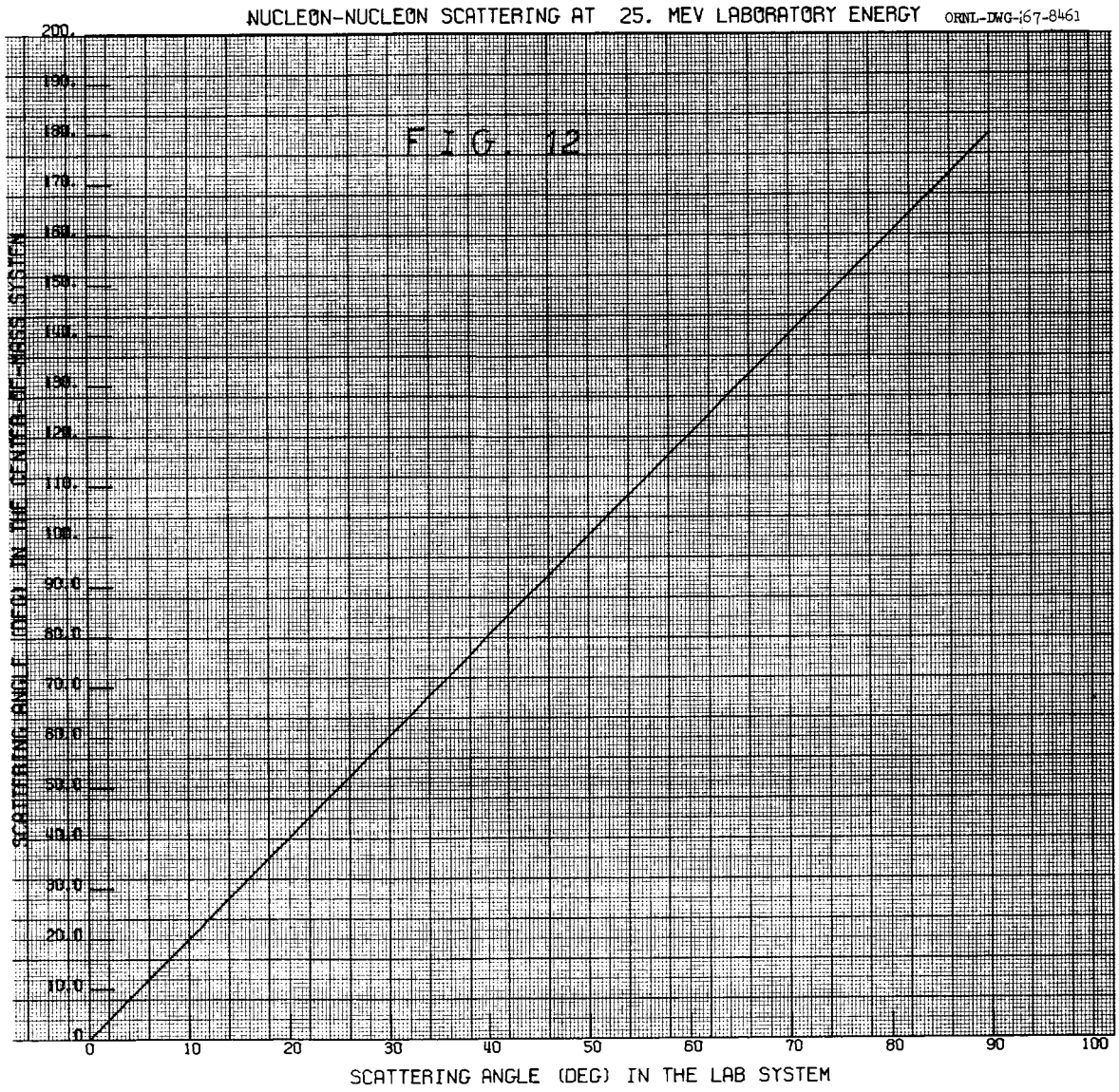
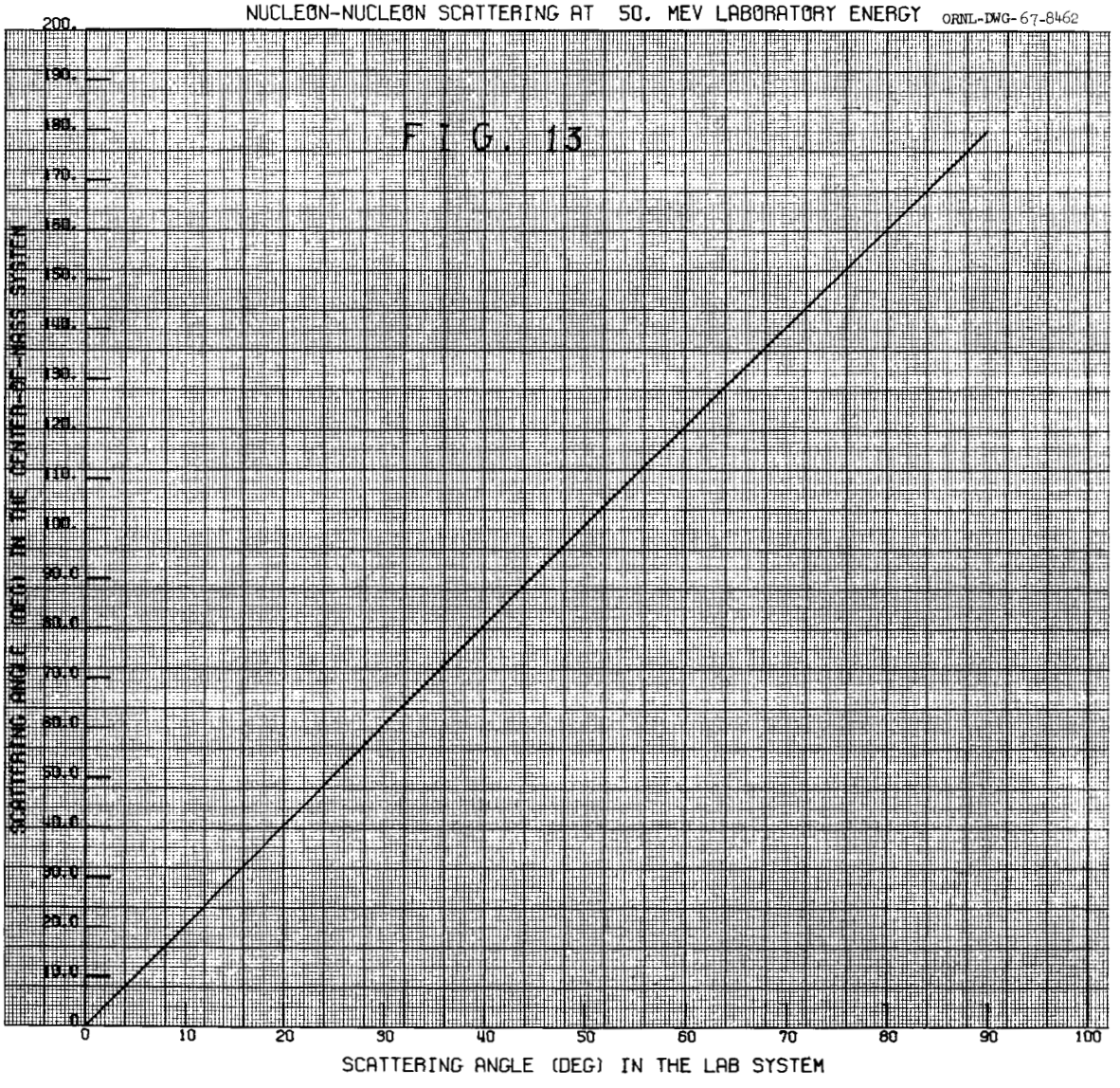
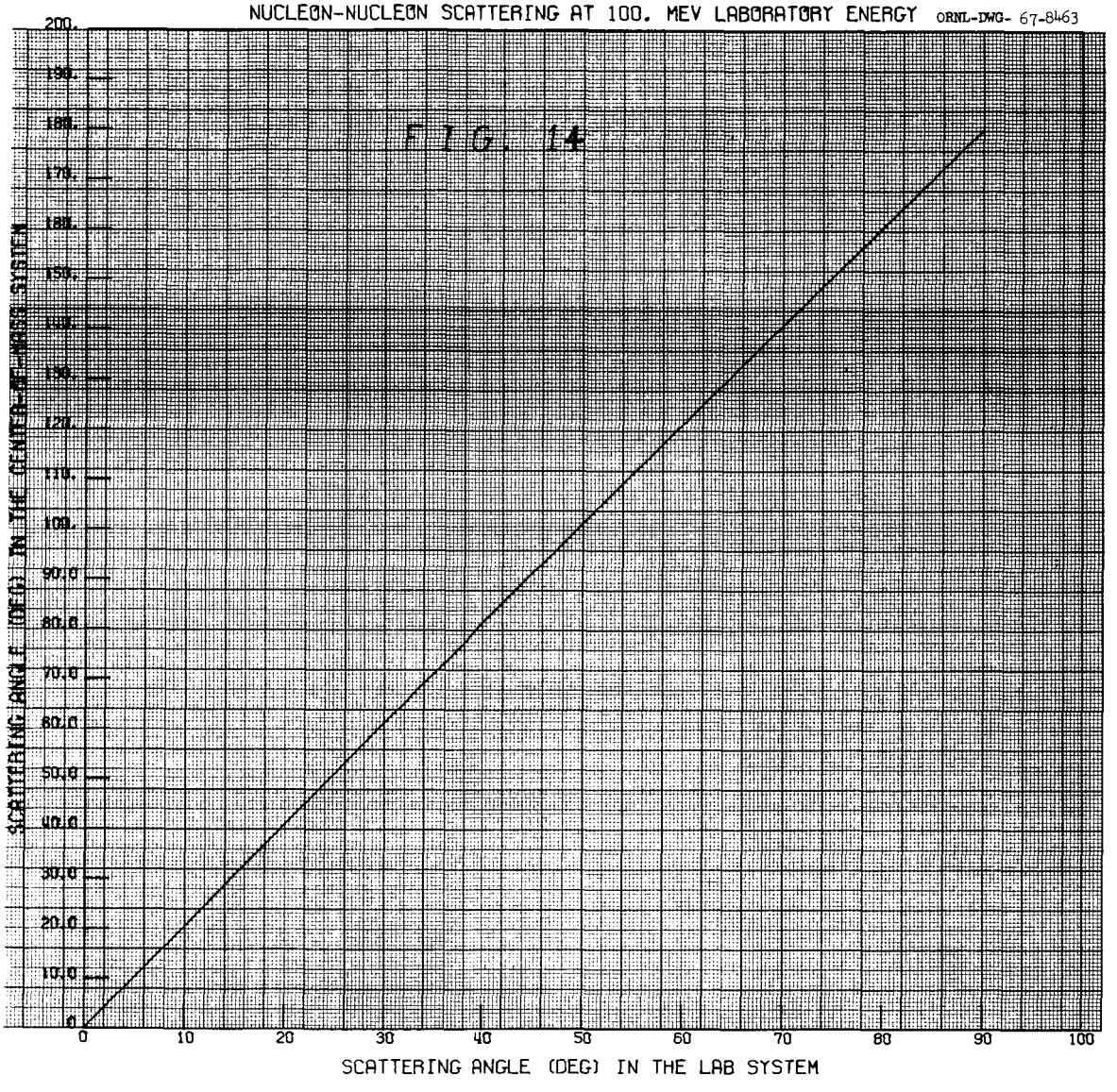

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, Θ
ORNL-DWG-67-8459
θ = 50°
θ = 55°
θ = 60°
θ = 65°
θ = 70°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000
400
360
320
280
240
200
160
120
80.0
40.0
0

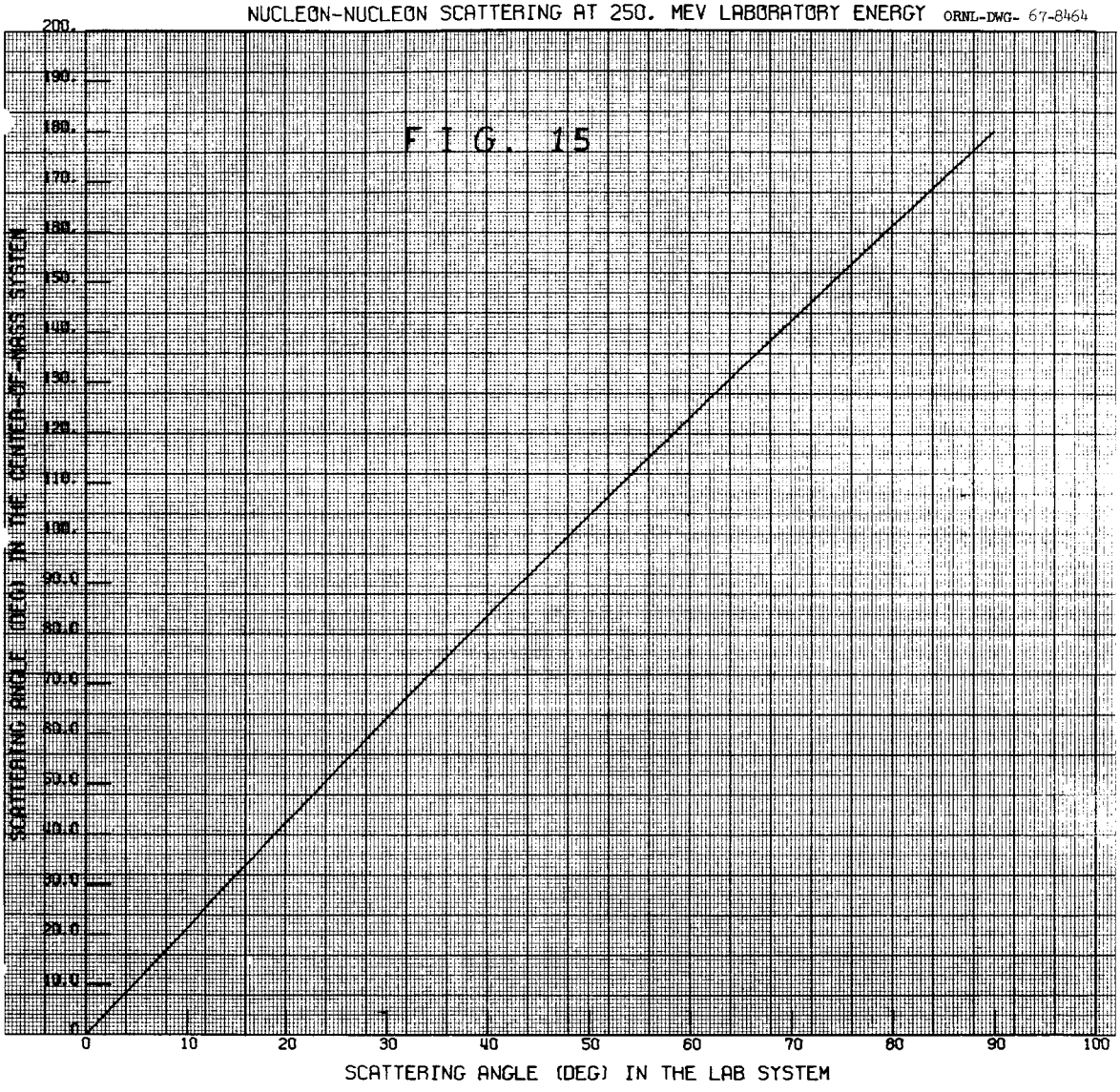
FIG. 10

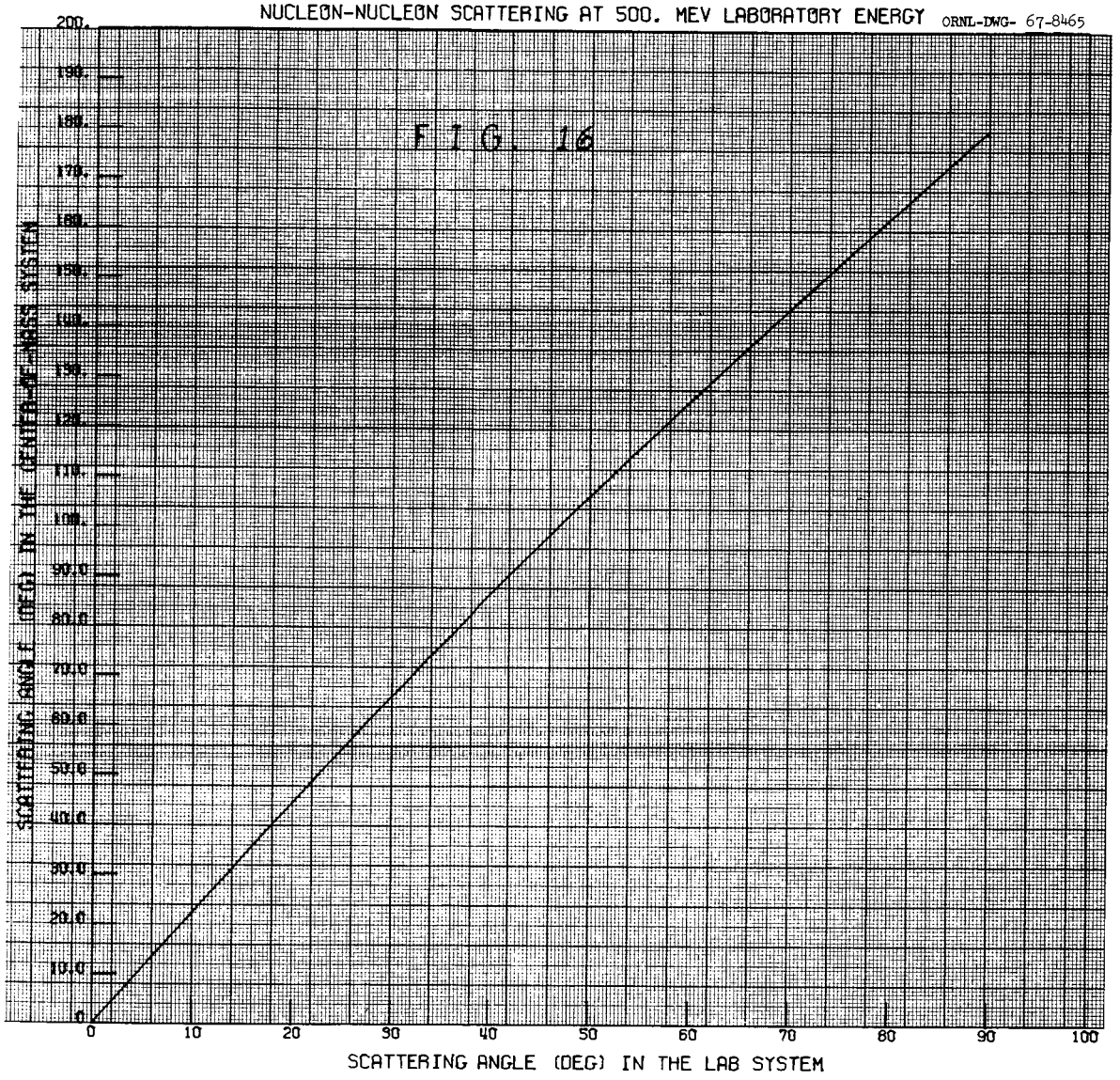

ENERGY OF SCATTERED PION VS INCIDENT PION ENERGY AND LABORATORY SCATTERING ANGLE, θ
ORNL-DWG-67-8460
FIG. 11
θ = 135°
θ = 125°
θ = 115°
SCATTERED PION ENERGY (MEV)
INCIDENT PION ENERGY (MEV)
0
250
500
750
1000
1250
1500
1750
2000
2250
2500
2750
3000
GRAPHIC CONTROLS CORPORATION BUFFA
No. 02

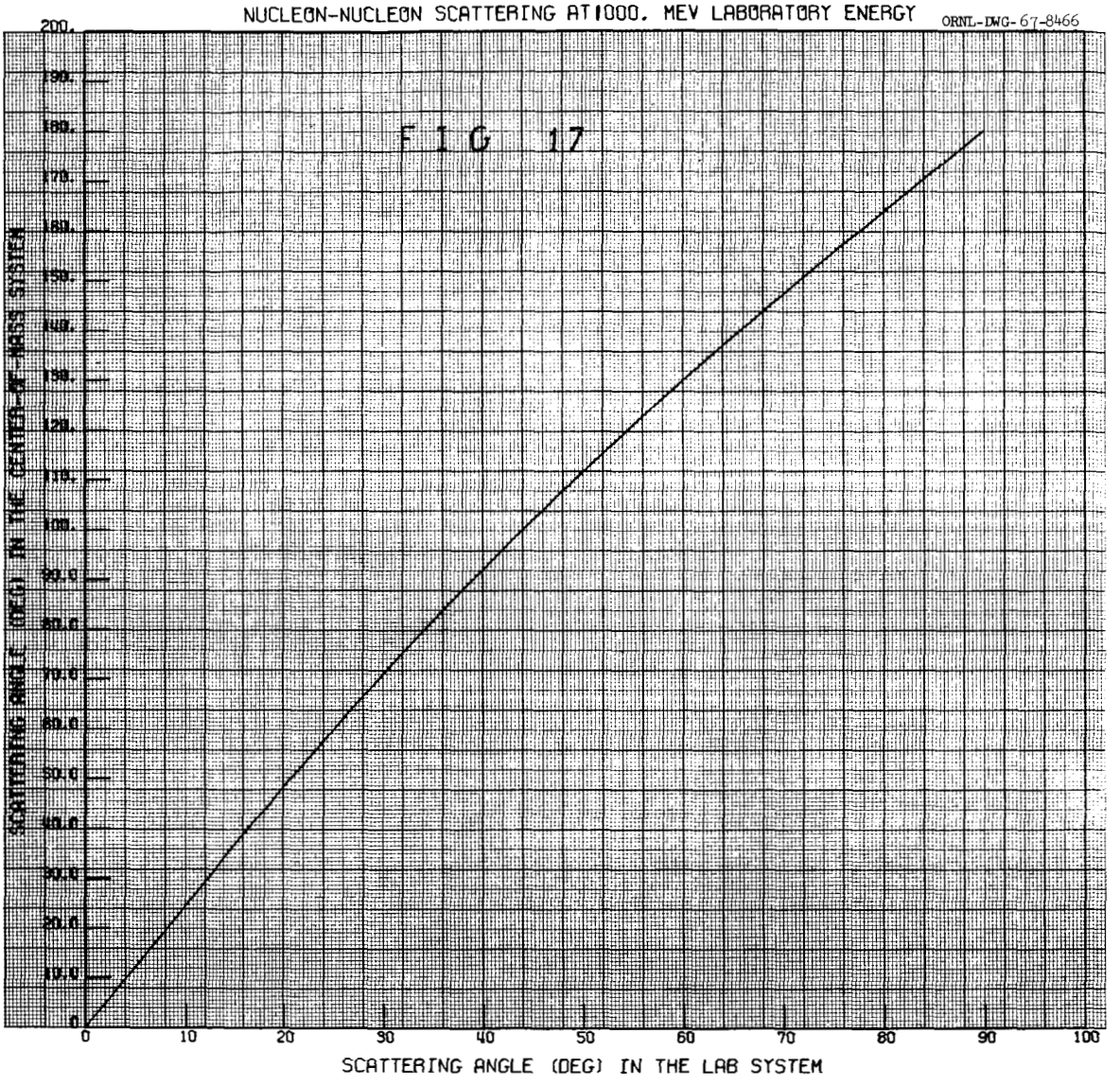

NUCLEON-NUCLEON SCATTERING AT 25. MEV LABORATORY ENERGY
ORNL-DWG-67-8461
FIG. 12
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100

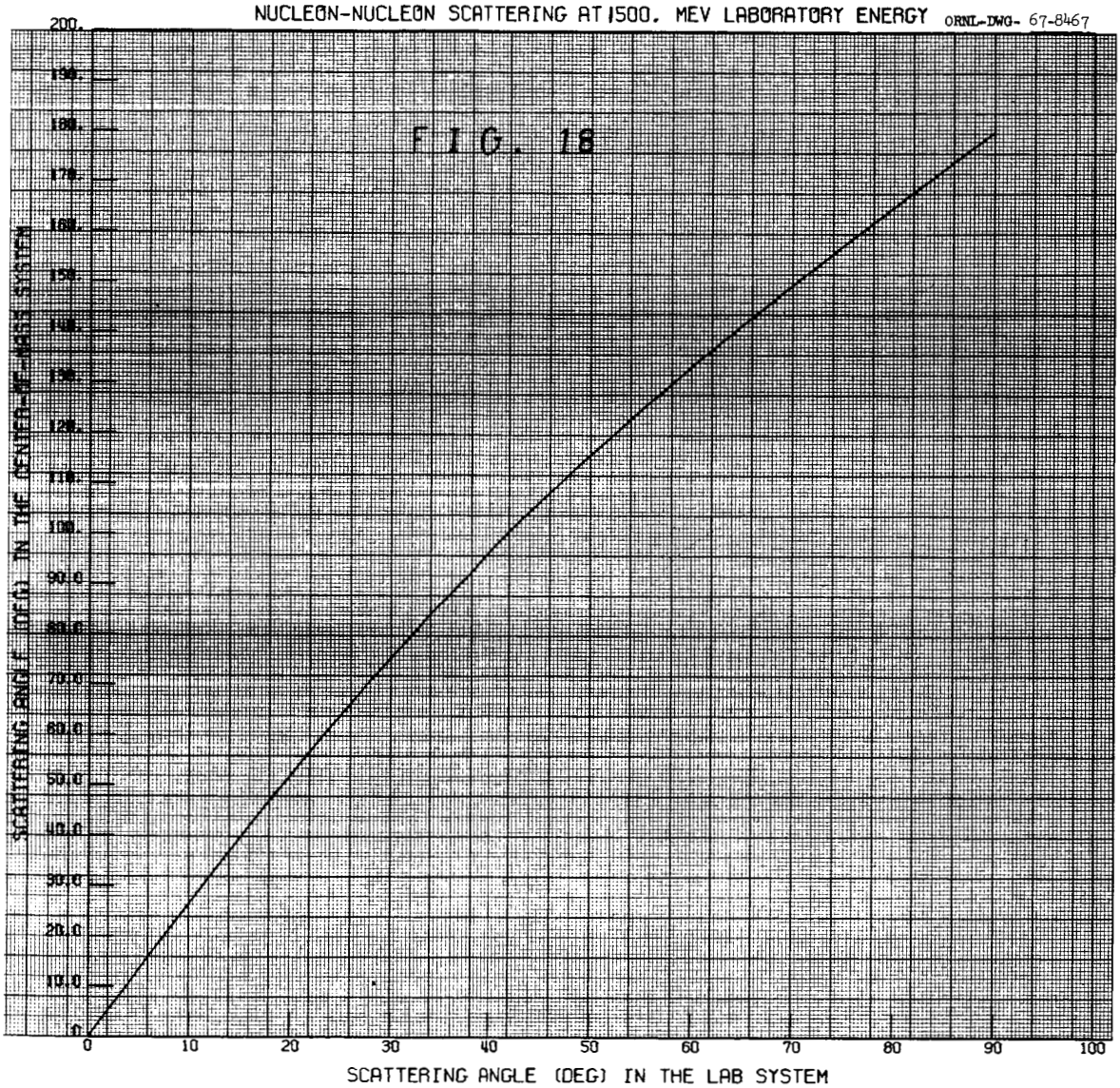

NUCLEON-NUCLEON SCATTERING AT 50. MEV LABORATORY ENERGY
ORNL-DWG-67-8462
FIG. 13
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100

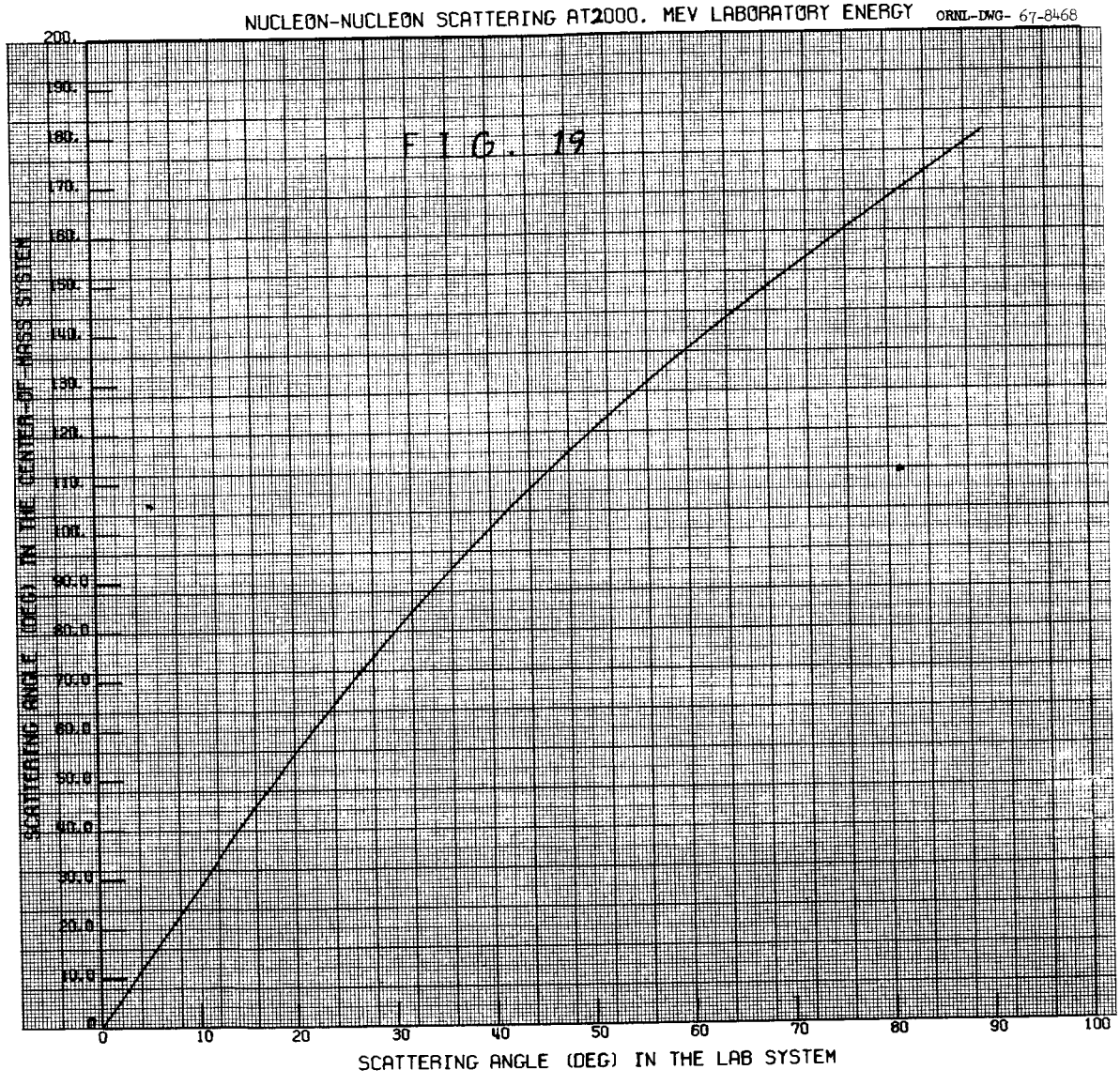

NUCLEON-NUCLEON SCATTERING AT 100. MEV LABORATORY ENERGY ORNL-DWG- 67-8463
FIG. 14
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
200.
190.
180.
170.
160.
150.
140.
130.
120.
110.
100.
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0
0
10
20
30
40
50
60
70
80
90
100
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM

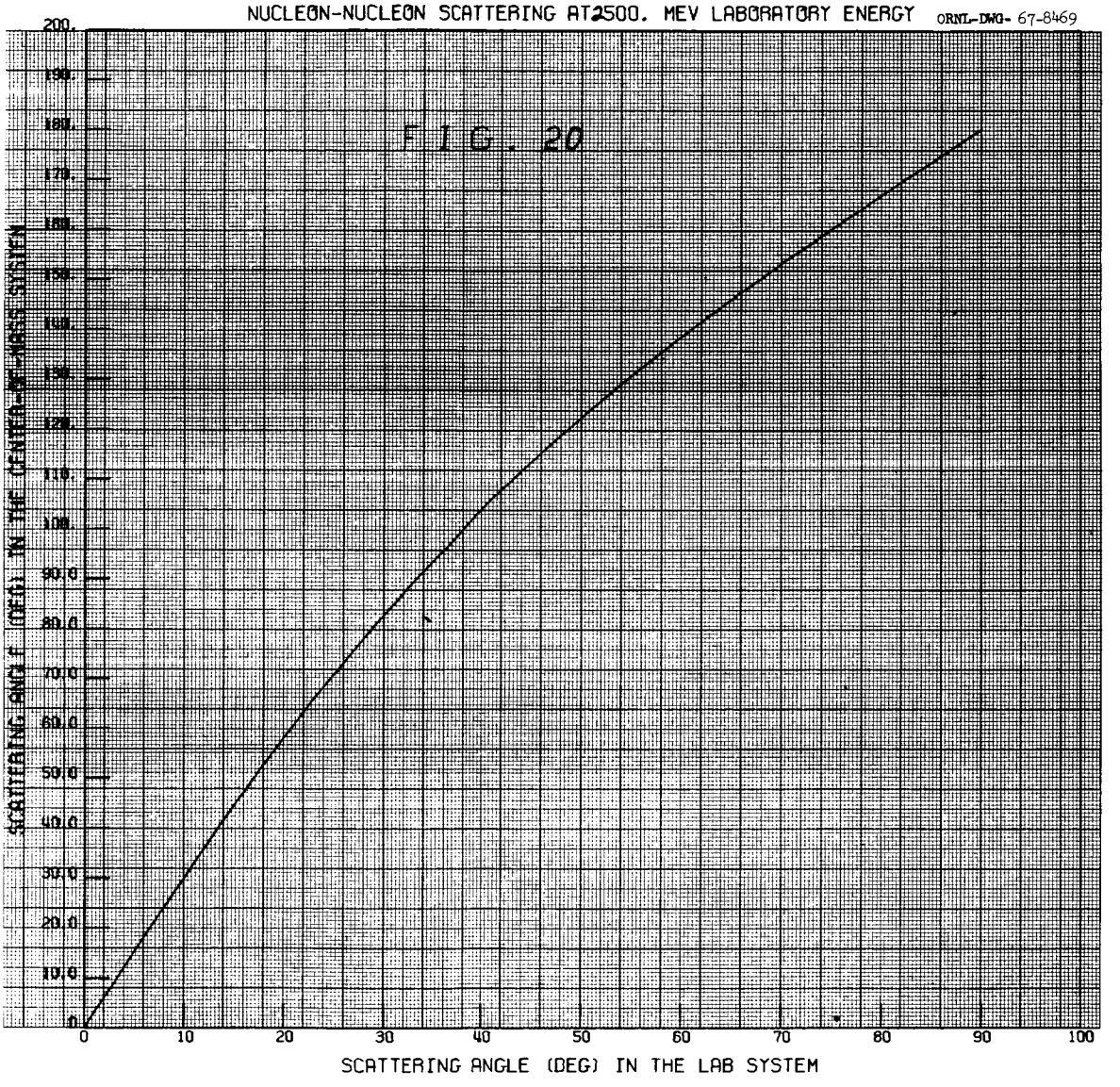

NUCLEON-NUCLEON SCATTERING AT 250. MEV LABORATORY ENERGY
ORNL-DWG- 67-8464
FIG. 15
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100

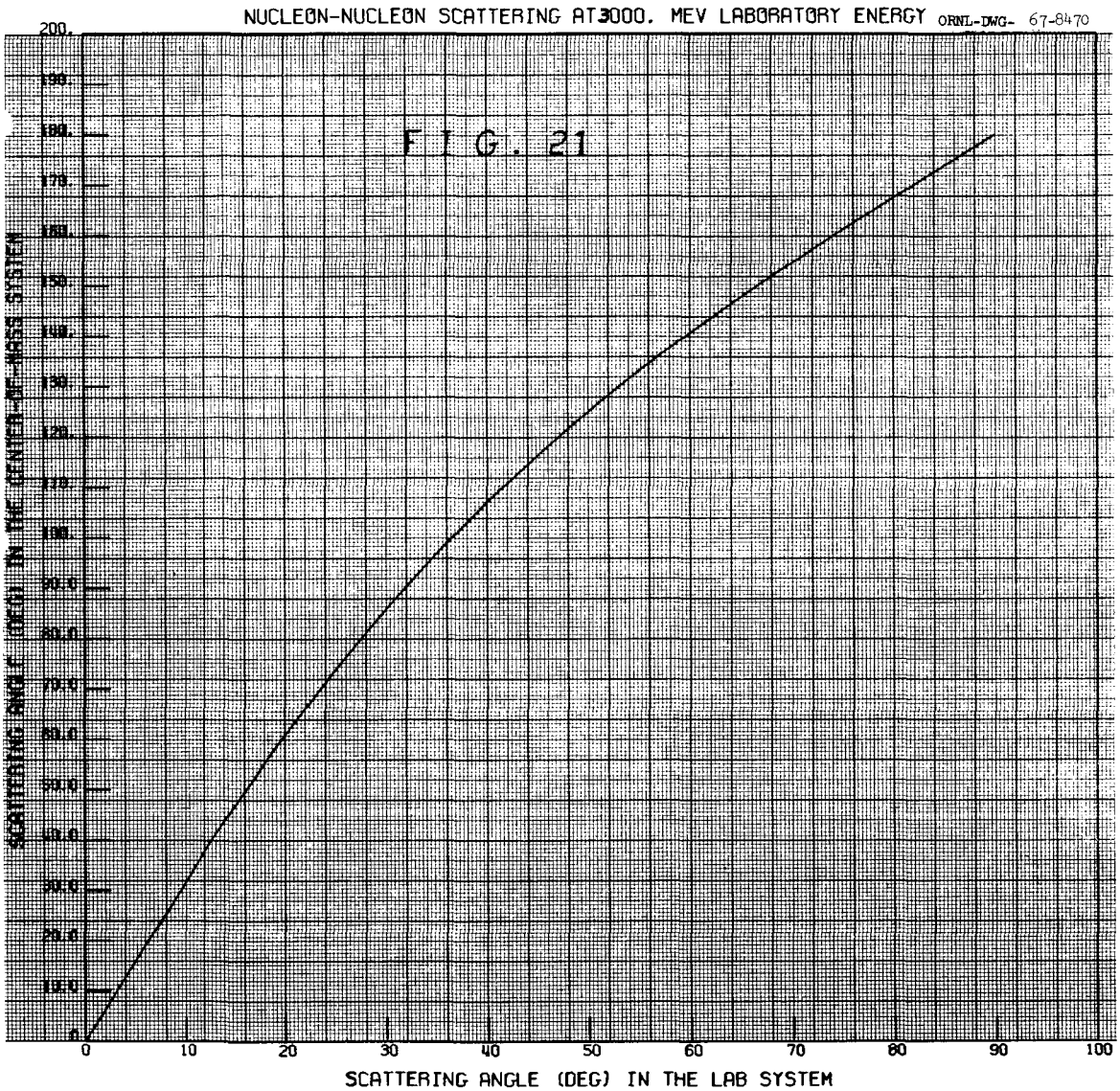

NUCLEON-NUCLEON SCATTERING AT 500. MEV LABORATORY ENERGY
ORNL-DWG- 67-8465
FIG. 16
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100

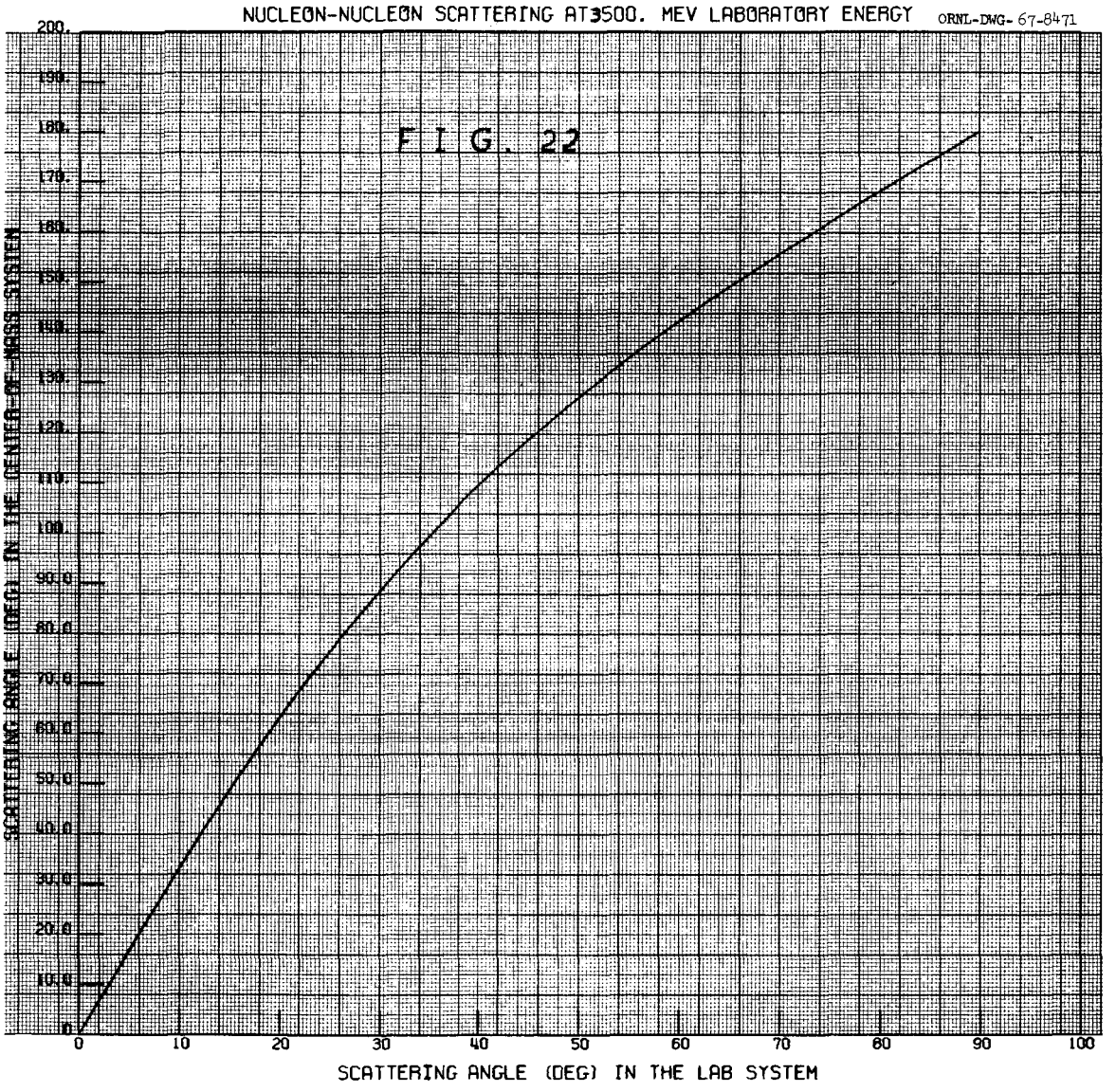

NUCLEON-NUCLEON SCATTERING AT 1000. MEV LABORATORY ENERGY
ORNL-DWG-67-8466
FIG 17
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100
200.
190.
180.
170.
160.
150.
140.
130.
120.
110.
100.
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0


NUCLEON-NUCLEON SCATTERING AT 1500. MEV LABORATORY ENERGY
ORNL-DWG- 67-8467
FIG. 18
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100
200.
0


NUCLEON-NUCLEON SCATTERING AT 2000. MEV LABORATORY ENERGY
ORNL-DWG- 67-8468
FIG. 19
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100


NUCLEON-NUCLEON SCATTERING AT 2500. MEV LABORATORY ENERGY
ORNL-DWG- 67-8469
FIG. 20
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100


NUCLEON-NUCLEON SCATTERING AT 3000. MEV LABORATORY ENERGY
ORNL-DWG- 67-8470
FIG. 21
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100


NUCLEON-NUCLEON SCATTERING AT 3500. MEV LABORATORY ENERGY
ORNL-DWG-67-8471
FIG. 22
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100
200.
190.
180.
170.
160.
150.
140.
130.
120.
110.
100.
90.0
80.0
70.0
60.0
50.0
40.0
30.0
20.0
10.0
0

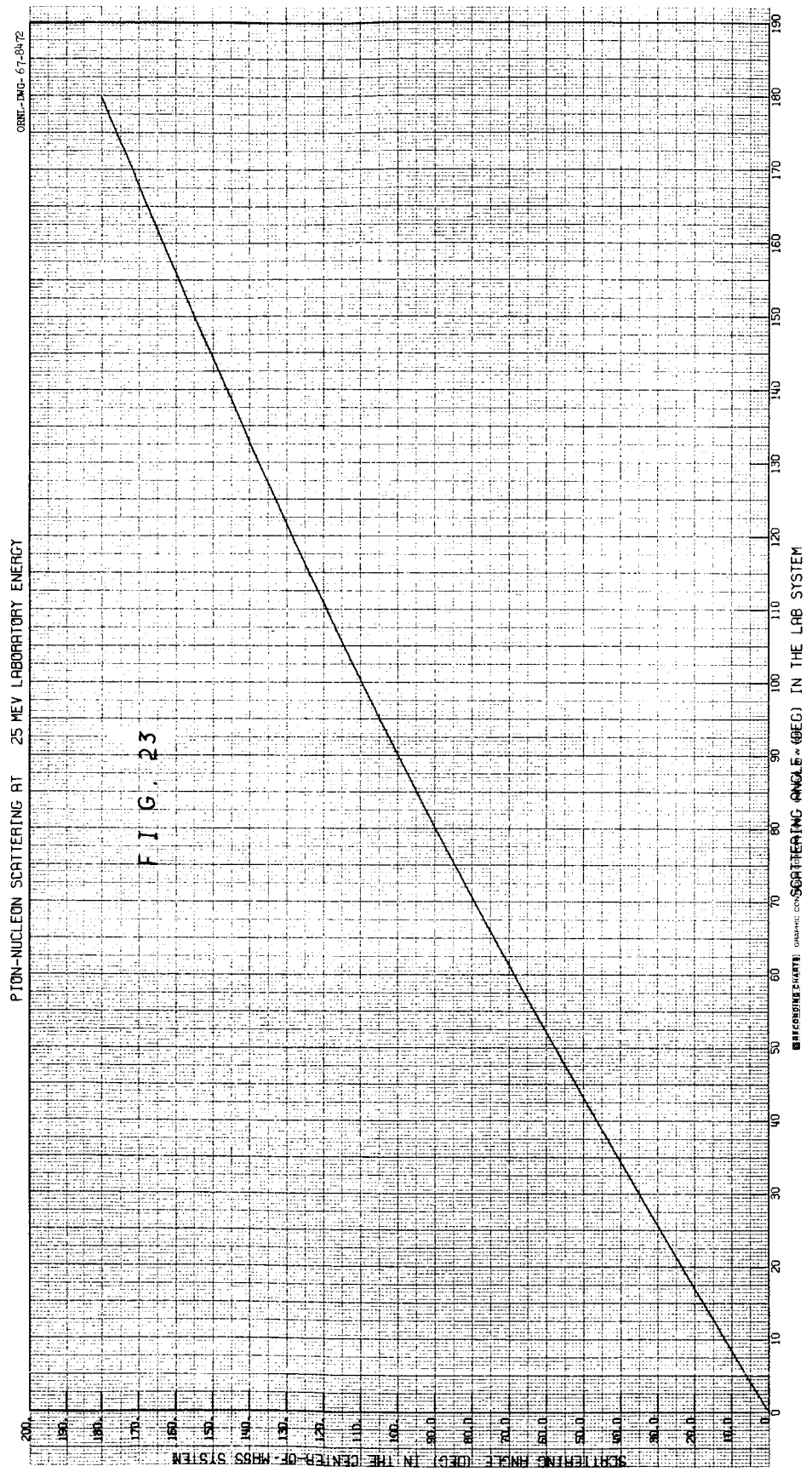
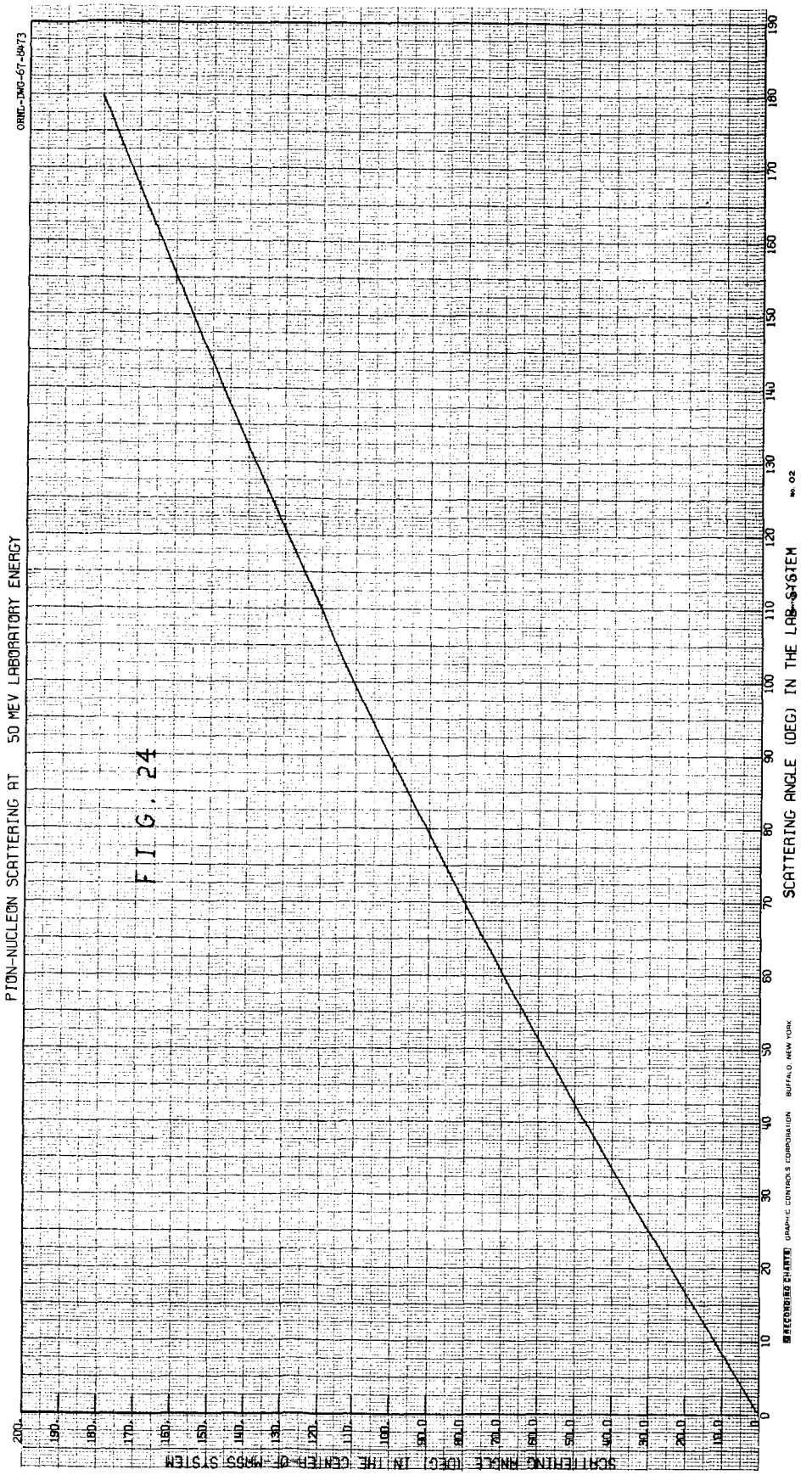

ORNL-DWG-67-8472
PION-NUCLEON SCATTERING AT 25 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM

FIG. 23


ORNL-DWG-67-6973
PION-NUCLEON SCATTERING AT 50 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
FIG. 24

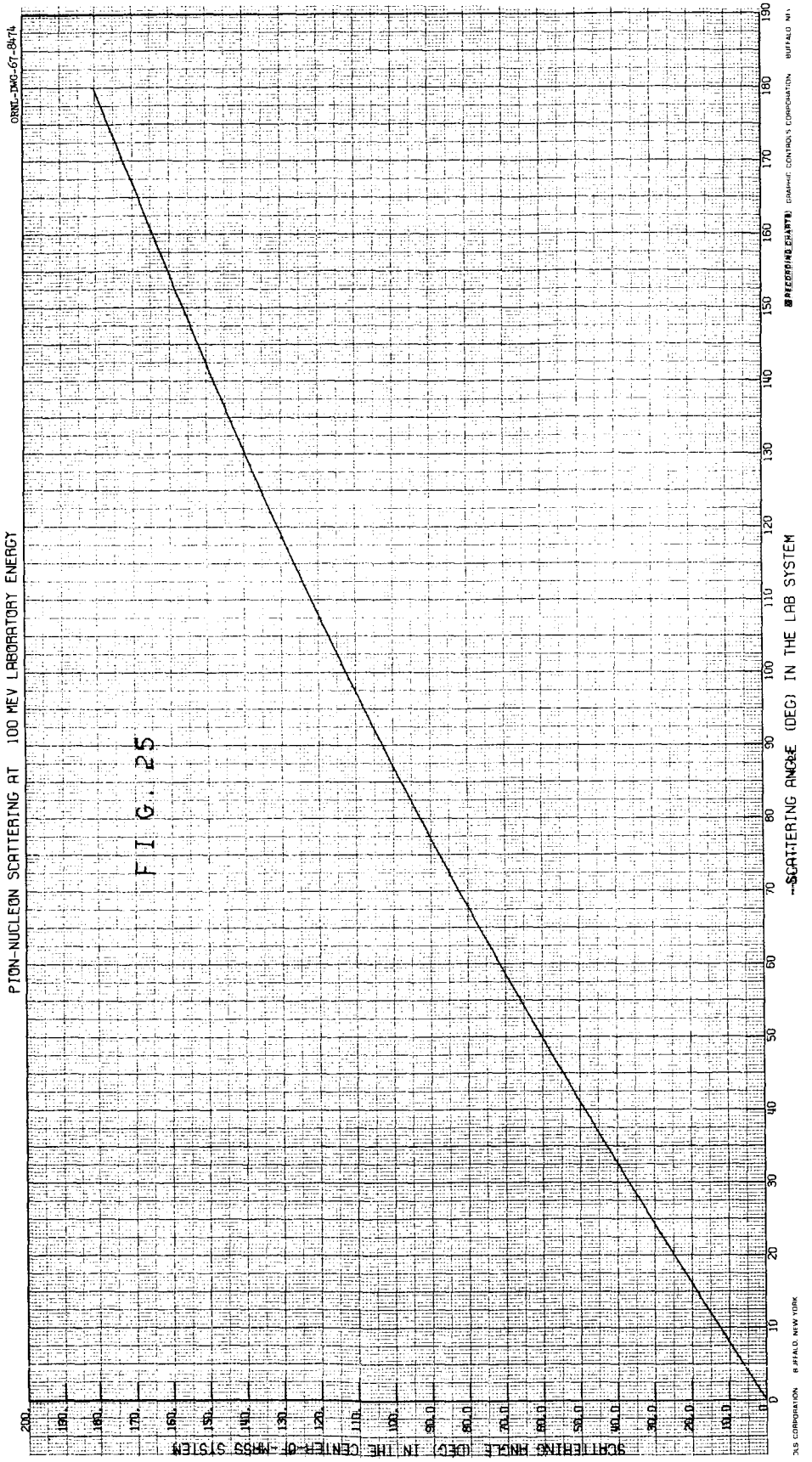
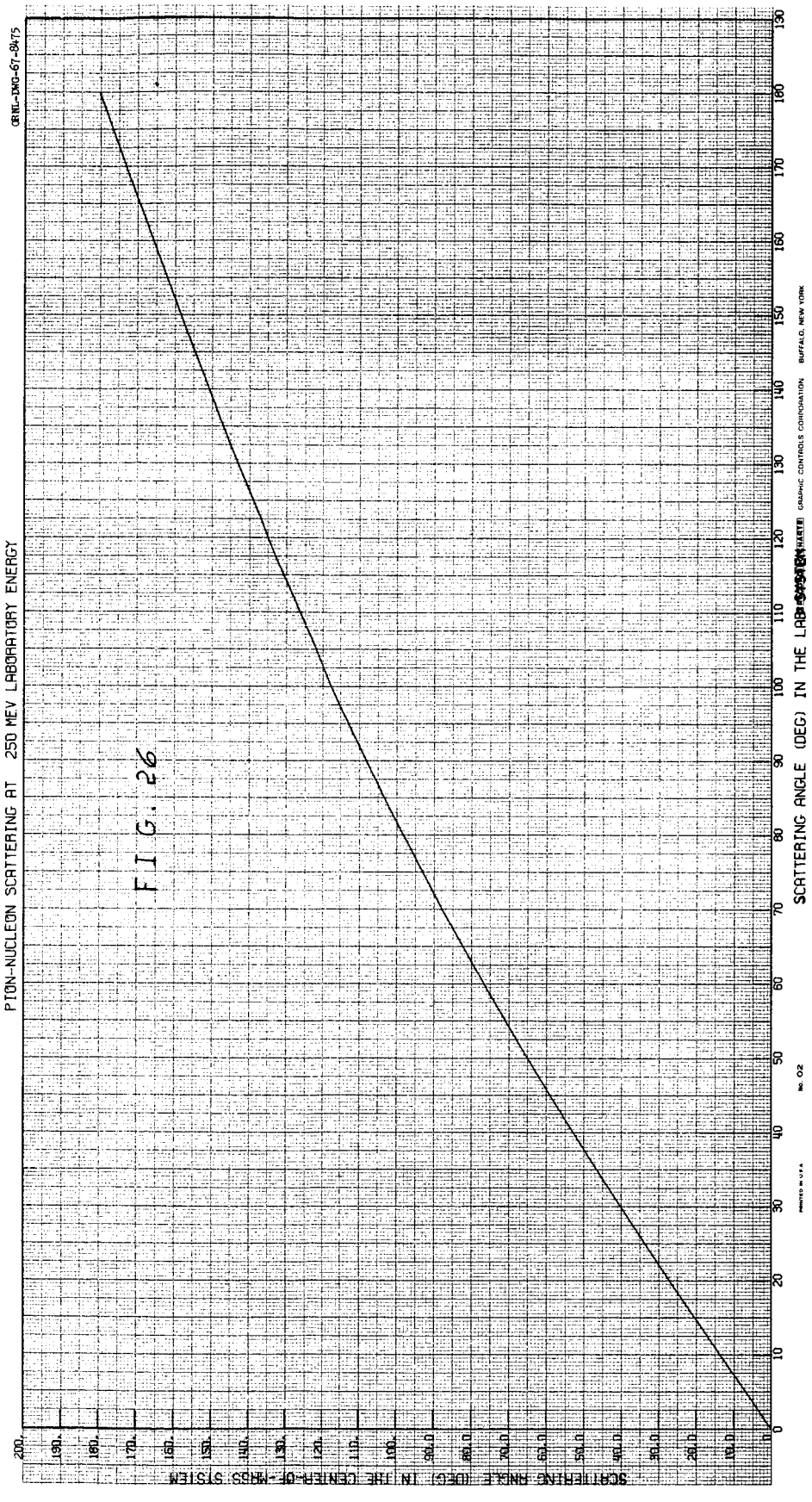

ORNL-DWG-67-8474
PION-NUCLEON SCATTERING AT 100 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.
110.
120.
130.
140.
150.
160.
170.
180.
190.
GRAPHIC CONTROLS CORPORATION BUFFALO, NEW YORK

FIG. 25


ORNL-DWG-67-9475
PION-NUCLEON SCATTERING AT 250 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE (DEG) IN THE LABORATORY SYSTEM
FIG. 26
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NO. 02

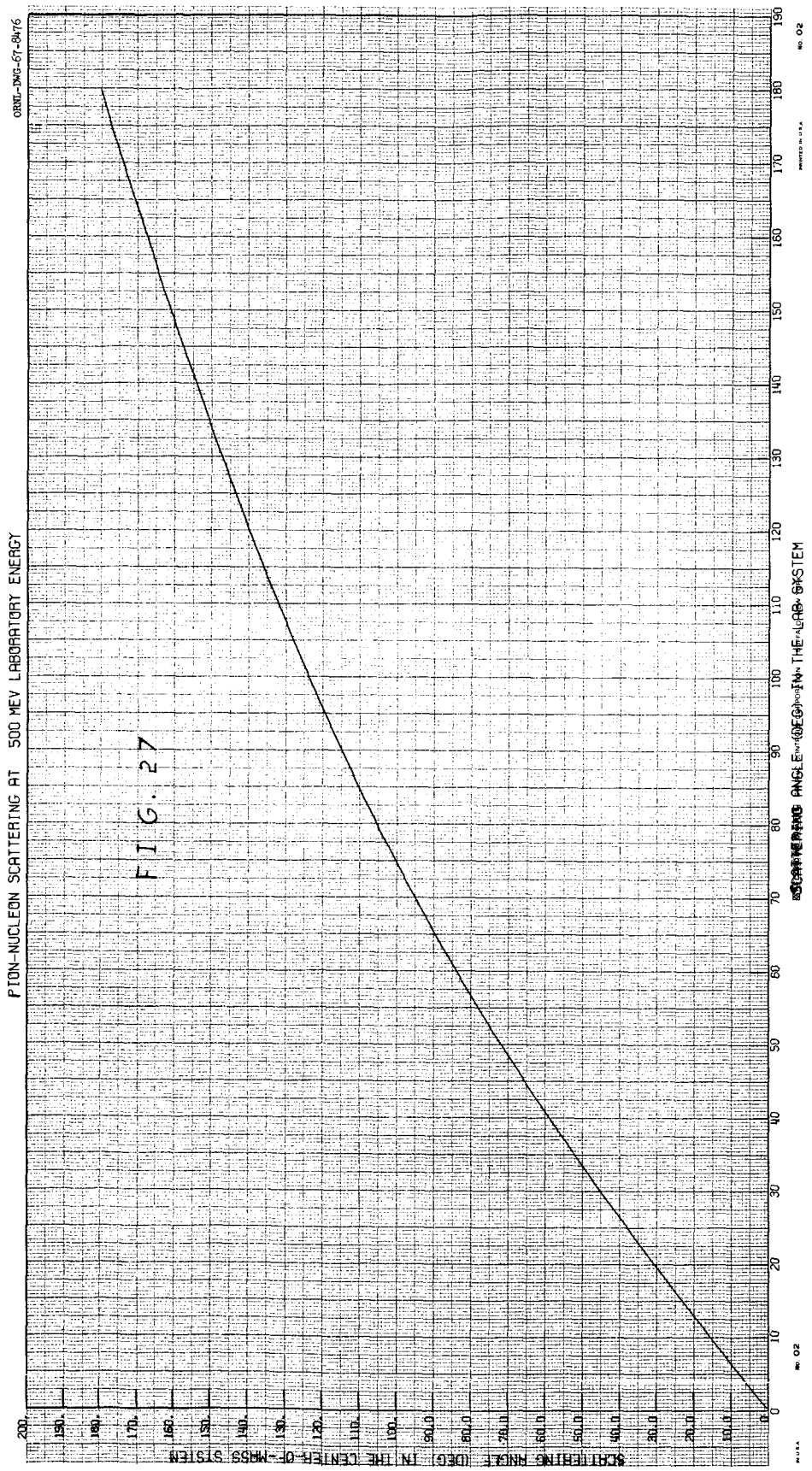
ORNL-DWG-67-8476
PION-NUCLEON SCATTERING AT 500 MEV LABORATORY ENERGY
SCATTERING ANGLE IN THE CENTER-OF-MASS SYSTEM (DEG)
SCATTERING ANGLE IN THE LAB SYSTEM (DEG)

FIG. 27

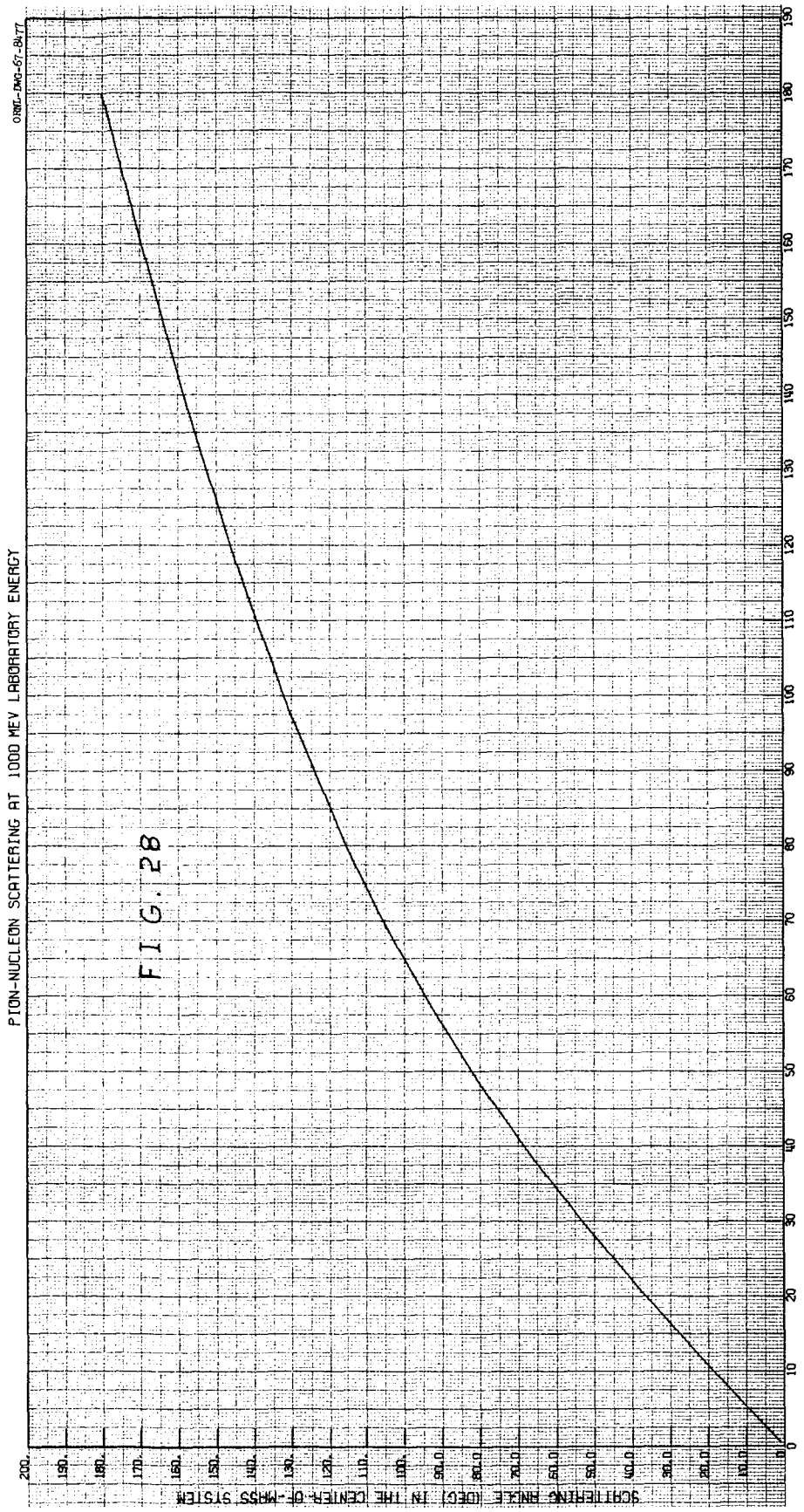
ORNL-DWG-67-8477
PION-NUCLEON SCATTERING AT 1000 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
SCATTERING ANGLE (DEG) IN THE CENTER OF-MASS SYSTEM
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.
110.
120.
130.
140.
150.
160.
170.
180.
190.
200.

FIG. 28

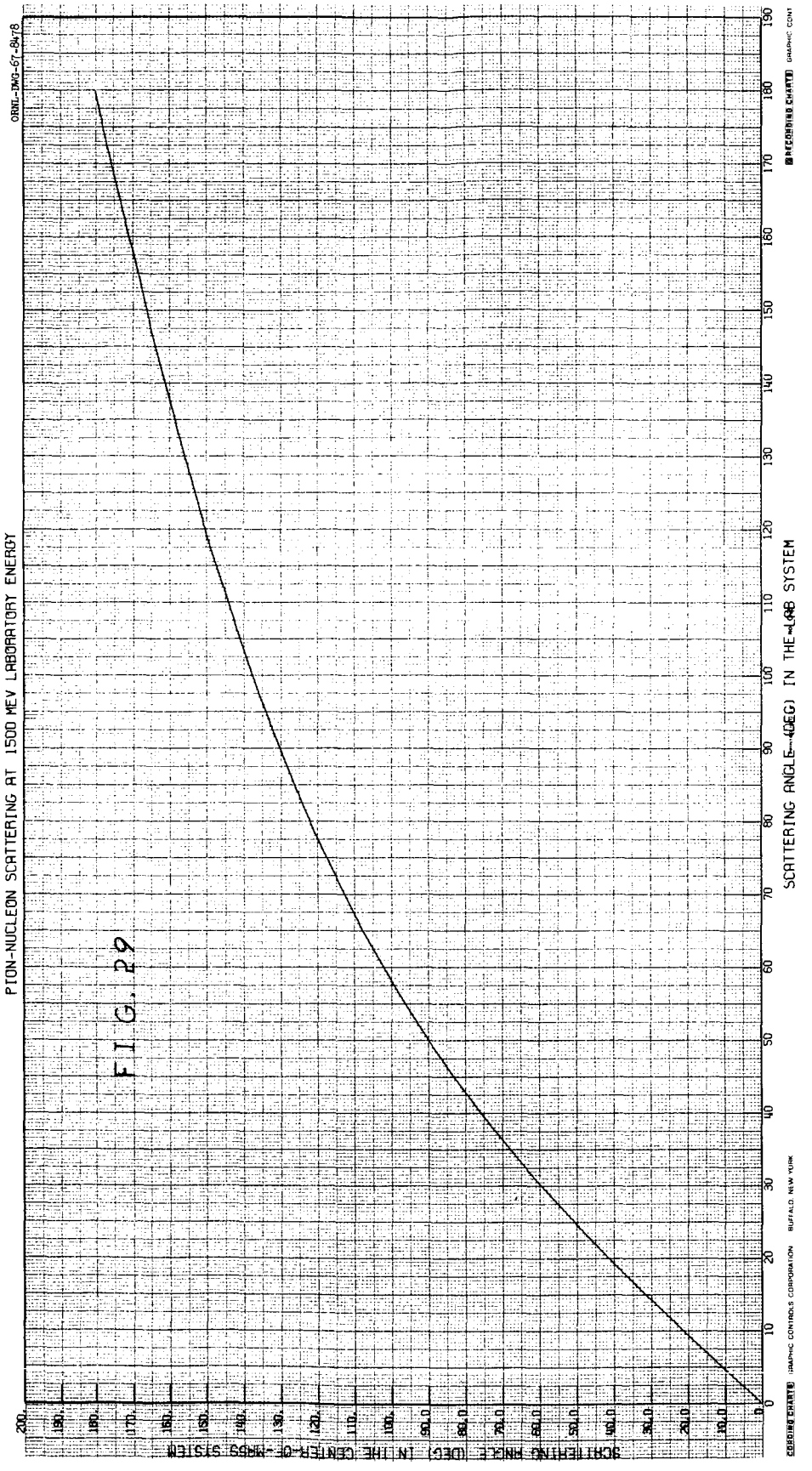
ORNL-DWG-67-8478
PION-NUCLEON SCATTERING AT 1500 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
0.0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.
110.
120.
130.
140.
150.
160.
170.
180.
190.
200.
GRAPHIC CONTROLS CORPORATION BUFFALO, NEW YORK
GRAPHIC CONT

FIG. 29

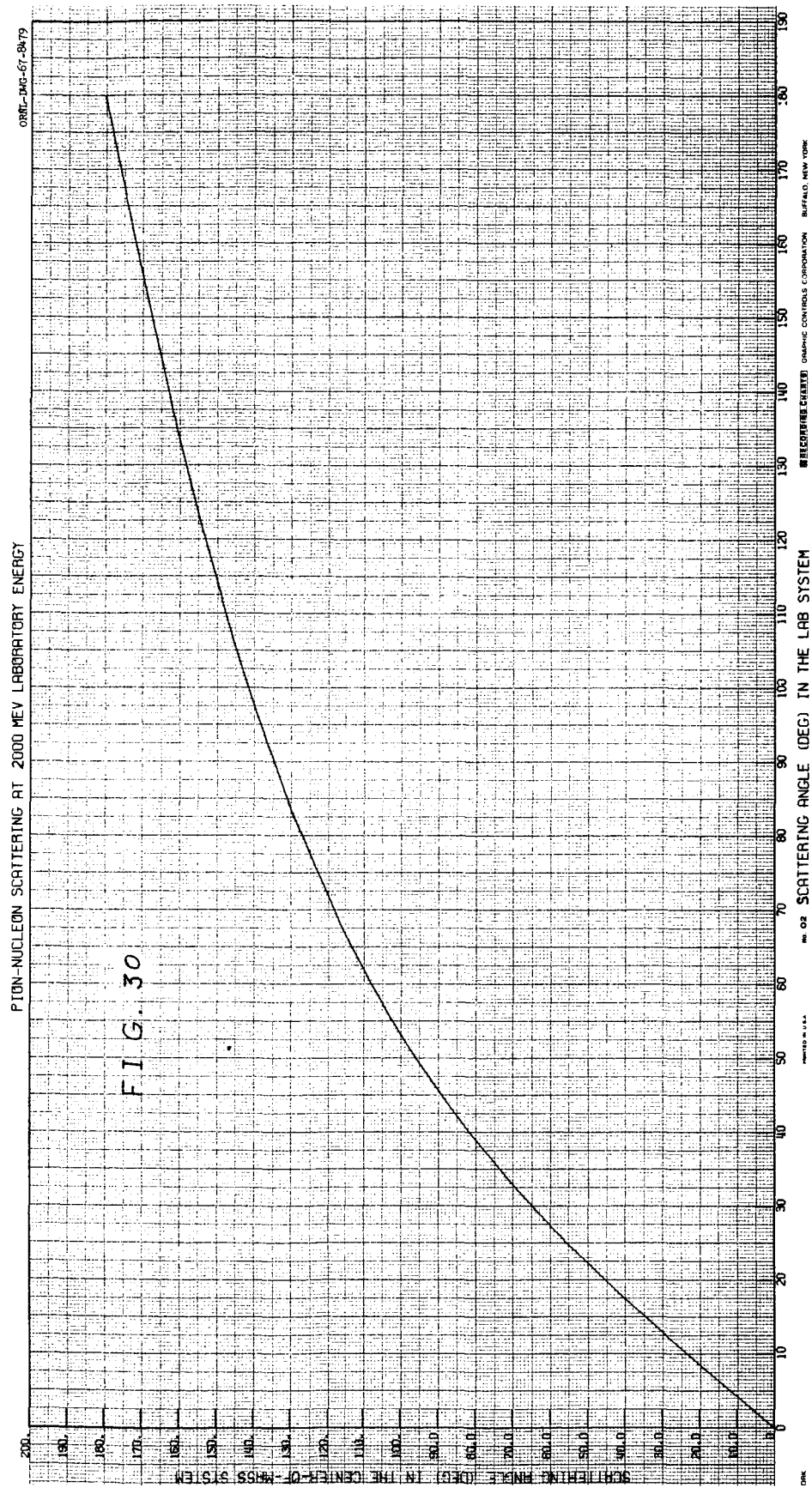
ORNL-DWG-67-8479
PION-NUCLEON SCATTERING AT 2000 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE LAB SYSTEM
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
GRAPHIC CONTROLS CORPORATION BUFFALO, NEW YORK
ORNL

FIG. 30

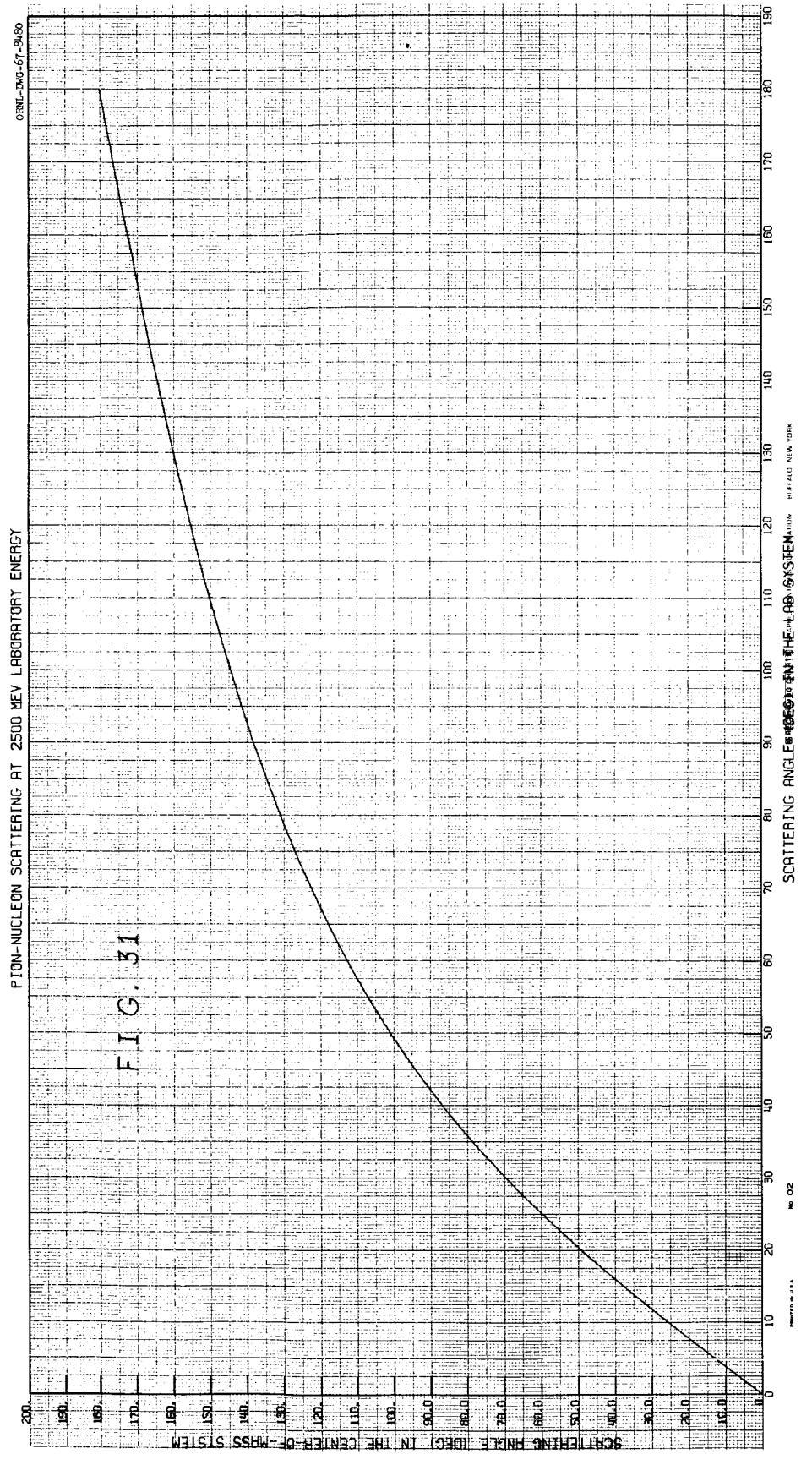
ORNL-DWG-67-8490
PION-NUCLEON SCATTERING AT 2500 MEV LABORATORY ENERGY
SCATTERING ANGLE (DEG) IN THE CENTER-OF-MASS SYSTEM
SCATTERING ANGLE IN THE LAB SYSTEM
BUFFALO NEW YORK
No 02

FIG. 31